Misspecified Politics and the Recurrence of Populism

Gilat Levy, Ronny Razin and Alwyn Young¹ This version: January 2021

Abstract: We develop a model of political competition between two groups that differ in their subjective model of the data generating process for a common outcome. One group has a simpler model than the other group as they ignore some relevant policy variables. We show that perpetual changes of power are a natural feature of this dynamic learning environment and that simple world views -which can be interpreted as populist world viewsimply extreme policy choices. Periods in which those with a more complex model govern increase the specification error of the simpler world view, leading the latter to underrate the effectiveness of complex policies and overestimate the positive impact of a few extreme policy actions. Periods in which the group with the simple world view implement their narrow policies result in subpar outcomes and a weakening of their omitted variable bias. Policy cycles arise, where each group's tenure in power sows the seeds of its eventual electoral defeat.

"Democracy is complex, populism is simple" (R. Dahrendorf, 2007)

1 Introduction

Voters differ not merely in their economic interests and preferences, but also in their fundamental understanding of the data generating process that underlies observed outcomes. Consequently, because they consider the same historical data through the prism of different models, even fully rational and otherwise similar voters can have persistent differences of opinion. In politics, such differences in model specification translate into differences in realized policy decisions when different groups are in power. The consequent interplay between world views, beliefs and policy can generate systematic correlations across observed data that sustain differing beliefs and biases.

¹Department of Economics, London School of Economics. For helpful comments we thank seminar participants in conferences and seminars in University of Pennsylvania, University of Chicago, Edinburgh University, LSE and Bocconi.

Indeed, understanding the implications of differing world views can shed light on an important aspect of populism. While the amorphous concept of "populism" has perhaps as many definitions as authors, the simplicity of populist world views are an important aspect of such movements. Motivated by the experience of populism in Latin America, Dornbusch and Edwards (1991) suggest that populism is "an approach to economics that emphasizes growth and income redistribution and de-emphasizes the risks of inflation and deficit finance, external constraints and the reaction of economic agents to aggressive nonmarket policies." Under this view, populist policies are motivated by world views that focus only on a subset of factors (for example, only short-run considerations) compared to a more complex macroeconomic model of growth and inflation suggested by experts and adopted by other political players.² The more recent incidences of populism in the western world seem to be centered on a simple ethos of "the people" versus the "elite".³ This new rhetoric centers on the "will of the people" which, as some recent papers argued, has to be simplified to capture the common ground of many.⁴ Similarly, many theories view the defining features of recent populism movements as anti-expert, anti-science and against the rule of law, all complex features of liberal well-functioning democracies.⁵ Anti-pluralism, anti-immigration and nationalist views espoused by populists also necessitate a simple definition of group identities.

In practice, when in power or in opposition, populist politicians often offer narrow and extreme solutions, sometimes to detrimental affect. Dornbusch and Edwards (1991) analyse the different stages, as well as the grave consequences, of populist economic reforms in Latin America. Penal populism often overemphasizes the importance of tougher legislation and police funding, ignoring other issues such as the complex intersections of economic inequality,

⁵See also in Sonin (2020) a discussion of how one feature of populist is to offer simple solutions to complicated problems, such as checks and balances.

 $^{^{2}}$ See also Guiso et al (2017) and Bernhardt et al (2019) who consider populist politicians who cater to voters' short-term interests and ignore the long-term consequences (e.g., when enacting protectionist trade policies).

 $^{^{3}}$ See Mudde (2004).

⁴See Sonin (2020) and the survey by Guriev and Papaioannou (2020). A recent literature analyses politicians speeches and party manifestos to compare the complexity of language used by populist politicians to others. Decasdri and Boussalis (2019) analyse a corpus of 78,855 utterances from the most recent Italian parliament and show that a change in allegiance from a populist to a mainstream parliamentary group increases a lawmaker's plenary spoken language complexity. Bischof and Senninger (2018) analyse a measure of complexity to assess the language of manifestos in Austria and Germany in the period 1945–2013. It shows that differences between parties exist and support is found for the conjecture about populist parties as they employ significantly less complex language in their manifestos. Chen, Yan and Hu (2019) compared Clinton's and Trumps' campaign speeches during the 2016 general election showing that Clinton used a more diverse vocabulary compared with Trump.

inequality in healthcare, opportunities, mental health issues and structural discrimination.⁶ A related and similar one-dimensional view of the world is behind more current populist views and suggested policies about immigration.

To focus on the implications of simplistic world views on politics, we consider political competition between groups that share the same interests and preferences over common outcomes but differ in their subjective model of the causes of these outcomes. Specifically, we consider the following dynamic model. The common outcome is a linear function of a set of relevant policy variables as well as a random shock. Everyone in the policy is interested in maximizing this outcome (subject to a resource constraint), but individuals differ in their subjective models of the relation between policy variables and the outcome. A subjective model is also linear and considers a set of policies to be relevant. We analyze a polity with a complex type, and a simple type: The simple type's subjective model consists a subset of the relevant policies that comprise the subjective model of the complex type. For example, while a complex subjective model may consider prevention of crime as best treated with a range of policies involving investment in policing but also in employment, education and welfare, a simple model may view crime as stemming from a single cause, lack of law and order due to inadequate police funding. In our dynamic model, both groups start with a prior and learn overtime, via the prism of their subjective models, about the parameters determining the effect of each policy variable they deem relevant.

We assume that political competition takes a simple form so that the group that wins is the one that has a higher intensity of preferences (that is, the group that is more keen on winning the election rather than letting the other side win). This group chooses its ideal configuration of policies which are then implemented with small "bureaucratic" noise. At every period the outcome is observed and both groups, consistent with Bayesian updating, use OLS to update their beliefs. Our model is then a social learning environment in which the group that takes an action is chosen endogenously, and proponents of both simple and complex solutions learn from the actual outcome delivered by themselves as well as by the rival group. Note that observations are not iid over time as learning and hence current policies depends on previous shocks.

Our key result is that the dynamic process converges to a unique steady state, characterized by two important features: it involves perpetual *political cycles*, as well as *extreme policies* advocated by the group holding the simple world view. We first show how the political process involves perpetual political cycles. When the complex govern and implement their

⁶Enns (2014) document how shifts in public opinion about the penal system has affected politicians to offer more simplistic policy prescriptions and has increased incarceration. Jennings et al (2017) show how similar penal populist trends affect policy in the UK.

broad policy agenda this increases the omitted variable bias of the simple group. The simple group believes that the complex group are wasting resources on irrelevant policies and they fully attribute the outcomes they observe to the few actions taken on the policy instruments they deem relevant. This increases the simple group's assessment of the likely effectiveness of a more decisive narrow policy agenda and mobilizes them in support of political candidates who will implement it. However, when the simple govern they produce systematically inferior results, as their extreme actions are revealed to be less effective than anticipated. This reduces the intensity of their political activism, thereby allowing the complex group to regain power. Thus, the economy suffers from inevitable political cycles and the recurrence of narrow and inefficiently extreme policies.

The group holding the simple world view observe their complex rivals invest in policies which they deem irrelevant. For example, if the outcome is crime prevention, the simple see that the complex invest in welfare schemes and social integration programs which they deem as irrelevant and wasteful. One possible interpretation for this is that the simple group believe that their rivals target public money to narrow special interests rather than for the "common good" of the majority group. This interpretation bodes well with the anti-elite theory of populism, ascribing to populist supporters the frustration with policies of the elite which they see as unhelpful or not benefiting the "people".⁷

A second feature of the dynamic process is that simple world views imply extreme policy prescriptions. While extreme policies often involve simple rhetoric, we show that having a simple world view in a social learning environment implies extreme policies and beliefs on all the policy variables considered relevant by the simple group. Specifically, the beliefs of the simple about the effectiveness of policy instruments converge to a multiple, larger than one, of the corresponding beliefs of the complex groups. This arises as the simple learn through the prism of their model both from their own policy choices but also from the policies implemented by the complex group. As a result, when in power, the simple implement a narrow and exaggerated version of complex policies. Indeed an additional frequent theme in the literature is that the policies of populist politicians are extreme, misguided and harmful to the very groups that support them (e.g., Dornbusch and Edwards 1991). Our framework provides an explanation for the recurrence of subpar outcomes that are supported by rational voters.

While our result is that regime change is inevitable, it is also hastened by negative shocks to the economy. When a negative shock arises, the intensity of the group in power falls

⁷As Mudde (2004) writes, "In the populist mind, the elite are the henchmen of 'special interests'.... in contemporary populism a 'new class' has been identified, that of the 'progressives' and the 'politically correct'... In the following decades populists from all ideological persuasions would attack the dictatorship of the progressives."

by more than that of the opposition group. This arises as the group in power actually implements its ideal policy and hence learns more precisely that such policy is not effective. This accords with the conventional wisdom that large negative shocks trigger populism but might also end its term.

Our paper complements the growing literature about populism by uncovering two aspects of the dynamic political process. First, we highlight a novel mechanism for political cycles when misspecified simple and complex world views are held by different groups in the electorate. In a model with rational individuals we show how the dynamics of learning through misspecified models and endogenous power shifts renders political cycles to be natural and inevitable.

Second we provide a rationale for why simple world views imply extreme and suboptimal policy prescriptions. In this sense, our paper adds to the literature of political-economy models of sub-optimal populist policies. Accordingly et al (2013) model left-wing populist policies that are both harmful to elites and not in the interests of the majority poor as arising from the need for politicians to signal that they are not influenced by rich rightwing interests. Di Tella and Rotemberg (2016) analyze populism in a behavioural model in which voters are betrayal averse and may prefer incompetent leaders so as to minimize the chance of suffering from betrayal. Guiso et al (2017) define a populist party as one that champions short-term redistributive policies while discounting claims regarding longterm costs as representing elite interests. Bernhardt et al (2019) show how office seekingdemagogues who cater to voters' short term desires compete successfully with far-sighted representatives who guard the long-run interests of voters. Morelli et al (2020) show how in a world with information costs incompetent politicians who simplistically commit to fixed policies can be successful.⁸ Our framework expands this literature by linking the pursuit of suboptimal policies to the bias created by a misspecified interpretation of outcomes under optimal policies.

Our theoretical contribution is to establish convergence in a learning environment with a misspecified model. Convergence of beliefs in such environments is not guaranteed, and is especially problematic with multidimensional state spaces (Heidhues et al 2018, Bohren and Hauser 2019, Esponda et al 2019, and Frick et al 2020). Our paper provides an example of how convergence can be proven in a model with multiple agents, a multidimensional state space and continuous actions. Specifically, we use noise in the implementation of policies to establish convergence in an OLS framework.

Interest in learning with misspecified models dates back at least to Arrow and Green (1973), with examples including Bray (1982), Nyarko (1991), Esponda (2008) and, most

⁸For more examples see the recent survey by Guriev and Papaioannou (2020).

recently, Esponda and Pouzo (2016) and Molavi (2019). Several recent papers feature interactions between competing subjective models that share features of our framework. Mailath and Samuelson (2019) consider individuals with heterogenous models who exchange beliefs sequentially once they receive a one-off (private) data and characterize conditions under which beliefs converge. Eliaz and Spiegler (2019) present a static model of political competition based upon competing narratives that draw voters' attention to different causal variables and mechanisms. They focus on a static equilibrium and on the possibility of "false positive" variables (which are not necessarily policy variables). Montiel Olea et al (2019), with auctions as a motivation, consider competition between agents that use simple or complex models to explain a given set of exogenous data and find that simpler agents have greater confidence in their estimates in smaller data sets and less confidence asymptotically. In our framework the endogenous data produced by actors with different specifications generates persistent biases and differences in beliefs that asymptotically keep both types politically competitive.

The paper proceeds as follows: Section 2 presents our basic framework, wherein voters differ in their beliefs regarding the possible determinants of common outcomes. Section 3 establishes the convergence to the unique steady state and explains the two key results of cycles and extremism. In Section 4 we discuss several extensions and modelling assumptions. In particular we discuss the relation between the unique equilibrium we characterize and the Berk-Nash equilibria of this model. An appendix contains all proofs not in the text.

2 The Model

The Economic Environment: We consider a common outcome $y \in R$ whose realization at time t is governed by the data generating process:

(II.1)
$$y_t = (\mathbf{x}_t + \mathbf{n}_t)'\boldsymbol{\beta} + \varepsilon_t$$

where \mathbf{x}_t and $\boldsymbol{\beta}$ are vectors of k policy actions in \mathbb{R}^k and associated parameters, and $\varepsilon_t \in \mathbb{R}$, a mean zero iid normally distributed random shock.⁹ We assume that all elements of $\boldsymbol{\beta}$ are non zero. The term $\mathbf{n}_t \in \mathbb{R}^k$ is a k-vector of policy noise which could be thought of as small policy implementation shocks. The characteristics of the steady state do not depend on adding noise to the model; we do so to insure convergence as we discuss in Section 3.5. The components of noise \mathbf{n}_t are iid with zero mean and diagonal covariance matrix $\sigma_n^2 \mathbf{I}_k$, and are independent of both the policy vector \mathbf{x}_t and the shock to outcomes ε_t . We add noise to

⁹We can generalise our results to allow for a constant term in the output function under some additional assumptions.

all relevant k policies, but alternatively we could add noise to only the set of policies that are implemented at each period and the results would be the same.

Although y is described as a single outcome, one can equally think of it as a weighted average of multiple outcomes that are influenced by \mathbf{x}_t .¹⁰ Below, we use bold letters to denote vectors and when it does not lead to confusion often drop the subscript t, writing \mathbf{x} , y, \mathbf{n} and ε .

Subjective Models: We assume that citizens are divided into two "types" based upon their subjective model about which of the unknown parameters in β can potentially be nonzero. We shall focus our analysis on the case where "complex" types (C) that believe all elements of β might be non-zero compete politically with "simple" types (S) whose model is misspecified, in that they exclude some relevant policies and are hence certain that for these policies, the elements of β are zero. We can easily extend our analysis to the case in which C's model is also misspecified, and to the case in which also S considers some relevant policies which are not considered by C (see Section 4.1). We assume that both groups know that ε is normally distributed.

Example 1 (Tackling crime: Social policies versus law and order): Let $y = \beta_1 x_1 + \beta_2 x_2 + \varepsilon$ denote an aggregate measure of welfare which is negatively related to the rate of crime. Assume that x_1 is the level of investment in policing and law and order, and x_2 is the level of investment in youth services, education/employment opportunities, or integration programs. Suppose that S believes that $\beta_2 = 0$ so that only law and order is relevant. In this case group S believes that investment in education, x_2 , is wasteful. This world view might come from a belief that crime is affected by individual characteristics and can only be influenced by deterrence. C on the other hand, believes that a combination of both policies is effective. The limit case when $\beta_2 \to 0$ would be the case where group S also has the correct model. We will come back to this example to highlight some of our key results.

In the general model group S has a set of $k_s < k$ policies that it deems relevant, while it believes that the effect of all other policies on y is null. We will use the subscript $i \in \{S, C\}$ to denote the group, where \mathbf{x}_i and $\bar{\boldsymbol{\beta}}_i$ denote the policy choice and the mean beliefs of group i. Unless otherwise specified all vectors of policies and beliefs will be k-vectors, where zeroes will be used for elements of the vector which are null. Specifically, \mathbf{x}_s and $\bar{\boldsymbol{\beta}}_s$ are k-vectors, with zeroes in all the elements pertaining to the $k - k_s$ policies that group S deems irrelevant. Below we assume linear utility; together with the linear formulation of y, this implies that

¹⁰If utility is a weighted average of *i* components each with $y_{it} = (\mathbf{x}_t + \mathbf{n}_t)'\boldsymbol{\beta}_i + \varepsilon_{it}$, then the outcome, parameters and error term in *II*.1 are simply the weighted average of those components.

only mean beliefs matter, and we henceforth denote the vector of mean beliefs at period t by $\bar{\beta}_{st}$ and $\bar{\beta}_{ct}$.

Although the subjective model of $i \in \{S, C\}$ is fixed, the beliefs of type $i \in \{S, C\}$ about the magnitude of the elements and in particular the expectation of these beliefs, $\bar{\beta}_{it}$ will evolve over time according to (Bayesian) OLS estimation. We use $\mathbf{H}_t = \mathbf{X}_t + \mathbf{N}_t$ to denote the $t \ge k$ history of desired policy and iid noise. As the error ε is independent of contemporaneous policy, period by period OLS updating involves the standard formula where each type focuses on their relevant policy regressors. We assume that prior beliefs are normally distributed; as our results are in any case asymptotic, normal beliefs of this sort can be justified by the observation of a long pre-history of policy, as under fairly general conditions the likelihood function determines the shape of the posterior (Zellner 1971).¹¹

Preferences and optimal policies: We model utility with the minimal structure that allows for a tractable presentation. Specifically, we assume the utility citizens derive from the common outcome is linear:

$$(II.2) \quad U_t(y_t) = y_t,$$

and that the choice of policies is subject to the budget constraint $\mathbf{x}'_t \mathbf{x}_t \leq R$, where R is some bounded, exogenously-given, resource. The constraint is formulated so that it allows us not to worry about the signs of the elements of $\boldsymbol{\beta}$ or \mathbf{x} .

Given the above, it readily follows that at any period, given some mean beliefs β_i for type $i \in \{S, C\}$, the optimal myopic policy solves

(II.3)
$$\max_{\mathbf{x}_i \in \mathbb{R}^{k_i}} \mathbf{x}'_i \bar{\boldsymbol{\beta}}_i + \lambda (R - \mathbf{x}'_i \mathbf{x}_i)$$

resulting in

$$(II.4) \qquad \lambda = \frac{1}{2} \sqrt{\frac{\bar{\beta}_i' \bar{\beta}_i}{R}}, \ \mathbf{x}_i^* = \frac{\bar{\beta}_i}{\sqrt{\bar{\beta}_i' \bar{\beta}_i}} \sqrt{R} \Rightarrow$$
$$\bar{y}[\mathbf{x}_i^*, \bar{\beta}_i] \equiv \mathbf{x}_i^{*'} \bar{\beta}_i = \sqrt{\bar{\beta}_i' \bar{\beta}_i} \sqrt{R}$$

While the solution to the Lagrangian problem is straightforward, we note here that given the constraint R, types which have more extreme parameter estimates, as measured by $\bar{\beta}'_i \bar{\beta}_i$, believe they know how to pursue more effective policies, as measured by their expected outcome when choosing their optimal policies, $\bar{y}[\mathbf{x}^*_i, \bar{\beta}_i]$, and consequently feel more constrained by the resource limitation R, as measured by λ .

¹¹Specifically, consider prior beliefs across relevant policies that are normally distributed with mean $\bar{\beta}_0$ and joint covariance matrix $\sigma_0^2 \mathbf{V}_0^{-1}$, while the prior probability density function on σ_0^2 is inverted gamma. We then define the pre-history such that $\mathbf{V}_0 = \mathbf{H}'_0 \mathbf{H}_0$ and $\bar{\beta}_0 = (\mathbf{H}'_0 \mathbf{H}_0)^{-1} \mathbf{H}'_0 \mathbf{y}_0$.

In each period political competition will determine which type chooses current period policies:

The political competition: We first define the notion of intensity of preferences. Let

(II.5)
$$I_i = \bar{y}[\mathbf{x}_i^*, \bar{\boldsymbol{\beta}}_i] - \bar{y}[\mathbf{x}_j^*, \bar{\boldsymbol{\beta}}_i]$$

where $\bar{y}[\mathbf{x}_j^*, \bar{\boldsymbol{\beta}}_i]$ is type *i*'s expected outcome when type *j* chooses their optimal policy. The intensity of preferences of type *i* is therefore the loss this type incurs from *j*'s ideal policy compared to her own ideal policy, given her subjective model. I_i does not necessarily equal $-I_j$ as beliefs differ across the two types. More specifically:

(II.6)
$$I_s = \mathbf{x}_s^{*'} \bar{\boldsymbol{\beta}}_s - \mathbf{x}_c^{*'} \bar{\boldsymbol{\beta}}_s,$$
$$I_c = \mathbf{x}_c^{*'} \bar{\boldsymbol{\beta}}_c - \mathbf{x}_s^{*'} \bar{\boldsymbol{\beta}}_c.$$

We assume that at any period t, the type that has the higher intensity of preferences wins the election, and then implements her ideal policy in that period (we focus then on myopic choices of policies and discuss strategic choices of policies in Section 4.2).

Below we construct a political competition model which rationalizes why intensity of preferences is an engine for power shifts. Assume that the polity consists of two equally sized groups, simple and complex, each a continuum. Each group is represented by a "citizencandidate" that runs in the election and if elected, implements the type's ideal policy.¹² Voting is costly, but citizens vote because they believe that with some (exogenous) probability p their vote will be pivotal.¹³ Consequently, a voter l of type i will vote (for their own representative) if the expected gain from the implementation of type i's optimal policies relative to those of type j exceeds voter l's cost of voting, c_l , i.e.:

$$(II.7) \quad pI_i > c_l$$

Assume that c_l is iid drawn from a distribution of voting costs G(c) and that the cost distribution is the same for both groups. Thus, the vote share that candidates of each type garner will be an increasing function of the intensity of their type. Consequently, the election is won by the candidate representing the type with the greatest preference intensity. The results below can be generalized to allow for unequal group sizes and different distributions. For example, the case of unequal groups implies the smaller group will require a certain margin of voting preference intensity to motivate its base enough to win an election.

 $^{^{12}}$ Given how we model voting decisions, it is easy to see that the presence of such candidates, offering voters of each type their ideal policy, will drive out all other policy platforms.

¹³For simplicity we are not modelling strategic voting, i.e., p is not determined endogenously in the model. The parameter p could be interpreted as the perception of voters about the probability they are pivotal in the election.

Before defining our equilibrium notion, we now characterize voters' intensity of preferences:

Lemma 1: The following are equivalent:

(i) The intensity of group i is greater than that of group j;

(ii) The magnitude of group i's belief vector is greater than that of group j, i.e., $\bar{\beta}'_i \bar{\beta}_i > \bar{\beta}'_j \bar{\beta}_j$;

(iii) Group i expects to achieve a higher outcome when in power than group j do when they are in power, i.e., $\bar{y}[x_i^*, \bar{\beta}_i] > \bar{y}[x_i^*, \bar{\beta}_j]$.

Intuitively, individuals with more extreme parameter estimates feel the resource constraint more keenly (as exemplified by the Lagrange parameter λ in *II*.4) and hence lose more from a sub-optimal movement away from their constrained choice. Hence the dynamic change of power in our model will be determined by the relative magnitude of beliefs of the two types.¹⁴ As $\bar{y}[\mathbf{x}_i^*, \bar{\boldsymbol{\beta}}_i] = \sqrt{\bar{\boldsymbol{\beta}}_i' \bar{\boldsymbol{\beta}}_i} \sqrt{R}$, intensity is also higher for the group that believes it can produce a higher level of output.

Dynamics: We consider then the following dynamic process:

1. In any period t, the winning type $i \in \{S, C\}$, chooses her ideal policy \mathbf{x}_{it}^* given her beliefs, $\bar{\boldsymbol{\beta}}_{it}$.

2. Given \mathbf{x}_{it}^* , $y_t = \mathbf{x}_{it}^* \boldsymbol{\beta} + \varepsilon_t$ is realized (and utility U_t gained). Both types update their beliefs using OLS. Mean beliefs evolve to $\bar{\boldsymbol{\beta}}_{j(t+1)}$, for all $j \in \{S, C\}$.

3. Type S wins the election at period t+1 iff its intensity is higher, that is, $\bar{y}[\mathbf{x}^*_{s(t+1)}, \bar{\boldsymbol{\beta}}_{s(t+1)}] > \bar{y}[\mathbf{x}^*_{c(t+1)}, \bar{\boldsymbol{\beta}}_{c(t+1)}]$. In the case of equal intensities, some tie breaking rule determines the winner.¹⁵

While the model of group S is misspecified, they use OLS estimation (consistent with Bayes rule) to rationally update their beliefs. Crucially, while the shocks are iid, the policies and hence regressors are not, as each type learns from the observed actions which themselves depend on the endogenously evolving beliefs.

Remark 1 (Anti-elite sentiment): Note that S observes that group C invests in policies that group S feels are irrelevant. In our model it is not necessary to assume that S knows

$$\bar{y}[\mathbf{x}_i^*, \bar{\boldsymbol{\beta}}_i] - \bar{y}[\mathbf{x}_i^* + \boldsymbol{\delta}, \bar{\boldsymbol{\beta}}_i] = \sqrt{\frac{\bar{\boldsymbol{\beta}}_i' \bar{\boldsymbol{\beta}}_i}{R}} \frac{\boldsymbol{\delta}' \boldsymbol{\delta}}{2}.$$

¹⁴To see the proof of Lemma 1, note that the gain in expected utility from pursuing an optimal policy \mathbf{x}_{i}^{*} versus an alternative policy in which a $k \times 1$ vector $\boldsymbol{\delta}$ is added to \mathbf{x}_{i}^{*} is given by $-\boldsymbol{\delta}'\bar{\boldsymbol{\beta}}_{i}$. Substituting using optimal policies and the fact that $-\boldsymbol{\delta}'\mathbf{x}_{i}^{*} = \frac{1}{2}\boldsymbol{\delta}'\boldsymbol{\delta}$, as both $\mathbf{x}_{i}^{*'}\mathbf{x}_{i}^{*}$ and $(\mathbf{x}_{i}^{*} + \boldsymbol{\delta})'(\mathbf{x}_{i}^{*} + \boldsymbol{\delta})$ equal R, we get that:

¹⁵The exact tie breaking rule is inconsequential.

that C has a different model. It can be the case that S believes that C is corrupt and invests in policies that do not benefit the general public but only a select group. This fits well with the anti-elite interpretation of populism ascribing to populist supporters frustration with policies of the liberal elite which they see as unhelpful or not benefiting the "people". For example, in relation to Example 1, they might view spending on welfare benefits or integration programs as wasteful and corrupt.

3 Perpetual Cycles and Extremist Populists

In this section we present Theorem 1, our main result, characterizing the unique steady state the dynamic model converges to. The steady state involves political cycles and extreme policies espoused and implemented by type S. To formalize the notion of political cycles, let θ_{jt} denote the share of time that $j \in \{S, C\}$ had been in power up to period t. Let β_s be the k-vector that agrees with the true parameters of β on all policies that group S deem relevant and has zero entries on all other $k - k_s$ policies and let $\tau^* = \sqrt{\frac{\beta'\beta}{\beta'_s\beta_s}} > 1$. We then have (for the proof see Appendices I-II):

Theorem 1: For sufficiently small σ_n^2 , the polity converges a.s. to a unique equilibrium in which: (i) **Political cycles**: $\theta_{st} \xrightarrow{a.s} \theta_s = \frac{1-\tau^*\sigma_n^2}{1+\tau^*}, \ 0 < \theta_s < 1$, (ii) $\bar{\beta}_{ct} \xrightarrow{a.s} \bar{\beta}_c = \beta$, (iii) **Colinear and extreme beliefs for** $S: \bar{\beta}_{st} \xrightarrow{a.s} \bar{\beta}_s = (\tau^*)\beta_s$.

We first discuss the intuition for the main findings of political cycles and extremism assuming that beliefs and the share of time that S is in power, θ_{st} , converge. We then provide a more technical discussion of how we prove convergence. Given that C has the correct model and given the policy implementation noise, it is easy to see that upon convergence, C will learn the true parameters of the model, and so $\bar{\beta}_{ct} \xrightarrow{a.s} \beta$.¹⁶

We focus then on the limit beliefs of S, $\bar{\beta}_s$, as well as on the limit values θ_s and θ_c (where $\theta_s + \theta_c = 1$). Let \mathbf{x}_i^* denote the optimal k-vector of policies of group $i \in \{S, C\}$ given their expected limit beliefs $\bar{\beta}_i$. For expositional reasons, we will henceforth consider the case of no policy noise, so that $\sigma_n^2 = 0$ (we reinstate the policy noise in Section 3.5 where we discuss convergence).

Given convergence, the OLS coefficients converge to satisfy the following equations:¹⁷

(III.1)
$$\theta_s \mathbf{x}_s^* (\mathbf{x}_s^{*'} \bar{\boldsymbol{\beta}}_s - \mathbf{x}_s^{*'} \boldsymbol{\beta}) + \theta_c \mathbf{x}_c^* (\mathbf{x}_c^{*'} \bar{\boldsymbol{\beta}}_s - \mathbf{x}_c^{*'} \boldsymbol{\beta}) = 0,$$

where $(\mathbf{x}_{s}^{*'}\boldsymbol{\bar{\beta}}_{s} - \mathbf{x}_{s}^{*'}\boldsymbol{\beta})$ and $(\mathbf{x}_{c}^{*'}\boldsymbol{\bar{\beta}}_{s} - \mathbf{x}_{c}^{*'}\boldsymbol{\beta})$ are the average mistakes that group S makes under

¹⁶Our results of cycles and extremism do not depend on the particular limit belief of C.

¹⁷This is the first order condition derived when minimizing expected squared mistakes.

her beliefs, when S is in power and when C is in power respectively. That is:

$$(III.2) \quad \mathbf{x}_{s}^{*'} \bar{\boldsymbol{\beta}}_{s} - \mathbf{x}_{s}^{*'} \boldsymbol{\beta} = \bar{y}[\bar{\boldsymbol{\beta}}_{s}, \mathbf{x}_{s}^{*}] - \bar{y}[\boldsymbol{\beta}, \mathbf{x}_{s}^{*}],$$
$$\mathbf{x}_{c}^{*'} \bar{\boldsymbol{\beta}}_{s} - \mathbf{x}_{c}^{*'} \boldsymbol{\beta} = \bar{y}[\bar{\boldsymbol{\beta}}_{s}, \mathbf{x}_{c}^{*}] - \bar{y}[\boldsymbol{\beta}, \mathbf{x}_{s}^{*}].$$

These expressions of average mistakes will play an important role in the intuition for our key results, which we now provide.

3.1 The Cycles of Populism

We now show how cycles must arise. When only one group is in power, let's say S, so that $\theta_s = 1$, this implies, from (III.1) and (III.2), that in the limit S's beliefs are such that they are not "surprised" anymore by the *average* output they produced, and so are not mistaken on average:

(III.3)
$$\bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_s^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*]$$

And trivially, C also predicts correctly the average output $\bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*]$. But note that C can do better than $\bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*]$. Its limit beliefs also explain what S does, but if it switches to its optimal policies given $\bar{\boldsymbol{\beta}}_c = \boldsymbol{\beta}$, namely \mathbf{x}_c^* , C can generate a higher output. Specifically, by shifting some resources from a narrow set of policies to the whole vector of policies, C uses the resources more efficiently and generates higher output. In other words,

(III.4)
$$\bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*] > \bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*] = \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_s^*]$$

This, by Lemma 1, implies then that C becomes more intense than S when S is assumed to hold power indefinitely. We then have a contradiction to this assumption, and so S must be replaced and cannot be in power for ever.

The exact same argument implies that when C is in power indefinitely, it is now that S becomes more intense. When $\theta_c = 1$, again, the beliefs of S (as well as those of C) converge to explain the average output produced by C; in the limit S is not surprised by what C is producing, with $\bar{\beta}_s$ solving $\mathbf{x}_c^{*'}\bar{\beta}_s = \mathbf{x}_c^{*'}\beta$. But given these beliefs, S realises that it can produce more if by shifting resources to its own narrow set of policies. Namely:

$$(III.5) \quad \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_s^*] > \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_c^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*] = \bar{y}[\bar{\boldsymbol{\beta}}_c, \mathbf{x}_c^*]$$

where the first inequality follows from the fact that \mathbf{x}_s^* maximizes output given $\bar{\boldsymbol{\beta}}_s$, and the other equalities follows from the observation that learning in the limit implies that both groups have beliefs that explain expected output.¹⁸

¹⁸Note that we assumed the true data generating process to be linear, but this is not important for the argument above, as whatever the process is, both groups will learn to explain the average output in the long term when one group is in power.

In other words, when one group is in power indefinitely, both groups learn to explain the average output it produces. But while the group in power also gets to implement its ideal policies given these beliefs, the group in opposition believes it can do better; this implies that it becomes more intense and power shift is inevitable. Thus long term dynamics must include political cycles. This can also be interpreted as a form of *incumbency disadvantage*. While the incumbent party implements its ideal policies given its beliefs, the opposition party finds the incumbent's policies wasteful, either as the incumbent invests in what it finds to be irrelevant policies (as is the case when S is in opposition), or as the incumbent invests too much in some policies (as in the case when C is in the opposition). In either case this leads the opposition party to believe it can induce a better outcome and makes it more motivated to replace the incumbent.

3.2 Fighting Crime: Cycles of Investment in Law and Order

To get some intuition about the cycles and long term dynamics, we now return to Example 1, where the true model is $y = \beta_1 x_1 + \beta_2 x_2 + \varepsilon$. We focus on crime prevention, y, with the policies being either investment in law and order, x_1 , or in social policies, x_2 (these could be integration/employment opportunities or social welfare policies). In this example S believes that $\beta_2 = 0$ so that only law and order, x_1 , is relevant. Recall that C's beliefs converge to the true values of β_1 and β_2 , and so we will focus on the evolution of the beliefs of S, which we denote by $\bar{\beta}_1$. Note that in this simple model $x_{1,s}^* = R$ for all beliefs of S, and so let us simplify further and assume R = 1.

In the criminology literature the populist tendency towards policies that are "tough on crime" has been termed penal populism. Enns (2014) documents the historical patterns of both voter attitudes towards law and order and policy outcome measures in the US. As a proxy for actual law and order policies, Enns (2014) uses changes in incarceration rates. The data shows that both attitudes towards "being tough on crime" and changes in incarceration have been decreasing from 1950-1970, increasing from 1970-mid 1990s and decreasing until 2010. While these trends are somewhat correlated with the level of crime, this relation exists even when controlling for crime rates and other economic variables. Similar patterns are shown in Jennings et al (2017) for the UK.

We now illustrate how the long term dynamics imply cycles of investment in law and order in our example. Whenever S is in power, they invest all resources in law and order, but over time become disillusioned with the benefits of higher police funding. In contrast, when C is in power, investment in law and order is lower (as it is accompanied with other policies), but over time in opposition S becomes convinced that tougher measures and more investment in policing are crucial. First, let us illustrate the political cycles result in this simple model. Note that when S is in power indefinitely, they have the true model to assess what they are doing; since they set $x_2 = 0$ their learning about x_1 is not biased. This implies that they will learn the true impact of law and order, β_1 . In this case, it is easy to see how C has greater intensity as it can produce a more effective crime prevention outcome by spreading resources efficiently on both policies. Alternatively, when C is in power forever, S's belief will suffer from an omitted variable bias and will be exaggerated and so $\bar{\beta}_1$ will solve:

$$(III.6) \quad \bar{\beta}_1 x_{1,c}^* = \beta_1 x_{1,c}^* + \beta_2 x_{2,c}^* \Rightarrow$$
$$\bar{\beta}_1 = \beta_1 + \beta_2 \frac{\beta_2}{\beta_1}$$

where we had substituted for the optimal policies of C. Again, as derived in the previous section, this implies that S develops greater intensity as S believes that substituting $x_{1,s}^*$ for $x_{1,c}^*$ will produce greater output on average, and so cycles must arise.

The implication of the political cycles result is that the belief of S must converge to satisfy equal intensity, as in any other case one group will be in power indefinitely. This pins down the (excessive) belief of S as follows:

$$(III.7) \quad y[\bar{\beta}_1, x_{1,s}] = y[\beta, x_c^*] \Rightarrow \bar{\beta}_1 = \sqrt{(\beta_1)^2 + (\beta_2)^2} > \beta_1.$$

Figure 1 below describes the asymptotic beliefs of S, close to the equilibrium belief defined above (note that these beliefs must be "sandwiched" between the limit beliefs that arise when each group is in power indefinitely). Close to the equal intensity belief, whenever the intensity of preferences of S is larger than that of C, it gains power and implements its ideal policy. But then, on average, S becomes disappointed in the outcomes it generates and moderates its belief towards the true β_1 . Simple voters are then systematically disappointed by the outcomes of their extreme investment in law and order. This leads to a gradual diminution of beliefs, until those with more complex views once again take power. But whenever S's intensity falls below that of C, and C gains power, S starts to inflate again the effectiveness of law and order. The surprising success of C's policies (which includes an array of other policies such as investment in education, integration and employment) gradually convinces simple voters of the value of law and order policies as they believe the success of Cstems from these policies only. This omitted variable bias that affects their belief increases their probability of voting in favour of populist politicians who advocate narrow and extreme solutions to complex problems. The equal intensity belief is then a basin of attraction for these dynamics.



Figure 1: The dynamics of S's belief

Note that these limit belief imply that S does not correctly conjecture the average output at each regime. In fact, the belief of S evolves to balance the mistakes in these conjectures *across* the two regimes. It is those mistakes that are the engine of the dynamics of power shifts in our model. While the above illustrate natural power changes over time, random shocks to the economy also play an important role in regime change, and we explore this in Section 3.4.

3.3 Simplicity implies Extremism

Many historical and contemporary examples of populistic politicians are considered to offer extreme rhetoric and policy prescriptions. Examples include left-wing economic populism in Latin America, or more recently anti-immigration and anti rule-of-law rhetoric (and policies) of right-wing populists in Europe and the US. Among the latter, examples abound, such as the policies of Victor Orban in Hungary, Poland's Law and Justice Party, the far-right Alternative for Germany (AfD), the National Front in France or Lega Nord in Italy, and the Republican party in the US under Trump's leadership.

In Example 1 above, we saw that the simple converge to believe that law and order is more effective compared with the belief of the complex type on these type of polices. This implies in general that their policy prescriptions for law and order are also more extreme. We now show that the relation between a simple model and extreme policy prescriptions holds in the general model when S considers more than one relevant policy. We show below that S's beliefs and policies will be more extreme on each of the policies which both groups find relevant. In particular, S's beliefs must be collinear with those of C on the relevant shared policies.

Extremism in our model is a result of the fact that S learns through the prism of her model both from her own policy choices but also from the policies implemented by C. If S is the only group in society, then as we saw above it would have a correctly specified model of how output is generated. Along with our assumption of some small implementation noise, S will then learn the true parameter values on the policies it considers. However, as power shifts are inevitable, the learning of S is substantially different as C is also in power, implying an omitted variable bias which as we show below, takes a specific form.

To understand the result of colinearity and extremism, suppose first that the steady state actions of S are *not* colinear with those of C. This implies, from (III.1), that the beliefs of S will evolve to fully explain the average output at each regime. Specifically,

$$(III.8) \quad \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_c^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*], \quad \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_s^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*]$$

These two equalities are linearly independent due to the fact that the policies are not colinear, and thus a solution exists. On an intuitive level, without colinearity there is enough variation in the data so that S will be able to correctly conjecture the average output delivered by each regime. But C, having the correct model, will also learn how to do this, which implies that S has greater intensity as:

(III.9)
$$\bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_s^*] > \bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}_c^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*] = \bar{y}[\bar{\boldsymbol{\beta}}_c, \mathbf{x}_c^*].$$

But this is in contradiction to our cycles result, which demands equal intensity. In other words, S cannot learn too much in equilibrium: Equilibrium policies must be collinear to limit the learning of S and specifically its ability to predict expected output at each regime. Instead, the belief of S will evolve to balance its prediction mistakes *across* the two regimes.

We can now fully derive the beliefs of S and show that S must hold more extreme beliefs than those of C. To see this, remember that in the long run the two types have equal intensity, i.e.,

(III.10)
$$\bar{y}[\bar{\boldsymbol{\beta}}_s, \mathbf{x}^*] = \bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*]$$

which by Lemma 1 implies that:

(III.11)
$$\bar{\boldsymbol{\beta}}_{s}^{\prime}\bar{\boldsymbol{\beta}}_{s} = \boldsymbol{\beta}^{\prime}\boldsymbol{\beta}$$

The colinearity result implies that $\bar{\beta}_s = \tau \beta_s$ for some τ . Plugging this into (III.11), we pin down the equilibrium degree of collinearity τ^* :

$$(III.12) \ (\tau^*)^2 (\beta'_s \beta_s) = \beta' \beta \Rightarrow \tau^* = \sqrt{\frac{\beta' \beta}{\beta'_s \beta_s}} > 1 \Rightarrow$$
$$\bar{\beta}_s = \sqrt{\frac{\beta' \beta}{\beta'_s \beta_s}} \beta_s$$

Thus, S is more bold in its policy prescriptions and so our model implies that simplicity implies extremism.

3.4 Dynamics of Power Shifts

Conditional on C's beliefs converging to the true parameters we have a unique equilibrium steady state. We now explore the comparative statics of the political cycles and how the true data generating process affects these dynamics.

First, we solve for the limit share of time that each group is in power. To solve for θ_s , we plug the expression for $\bar{\beta}_s$ from (III.12) in the OLS condition (III.1), where $\bar{\beta}_s$ is also required to explain mistakes across the two regimes. Noting that $\bar{\beta}_c = \beta$, we then get:

$$(III.13) \quad \theta_s = \frac{1}{1+\tau^*},$$

where it is easy to see that θ_s is lower when τ^* is higher. The collinearity parameter measures the relative importance of the parameters *not* considered by *S*. Therefore we have:

Observation 1: The more important are the policy variables that S ignores, the more extreme are S's belief, and the less time it spends in power.

Intuitively, to generate more extreme beliefs in equilibrium, S needs to suffer from a higher omitted variable bias, which arises when C is in power more often. Thus, political cycles must result in just enough omitted variable bias to equate intensity. This implies that if Sis extremely wrong to ignore some policies, so that τ^* is very large, then it spends very little time in power, but when it does, its policies are very biased. Alternatively when S is almost correct, τ^* is close to 1, and θ_s is close to a half.

The deterministic average power sharing θ_s captures the systematic changes of power. This results from the fact that the beliefs of S must balance the "mistakes" in its predictions across the two regimes. Thus, S is continuously "surprised" by its prediction for the average output for *each* regime. As in Figure 1, when they are in power, S is surprised that its policies have underperformed, as the true average outcome, $\bar{y}[\boldsymbol{\beta}, \mathbf{x}_s^*]$, is strictly lower than their expected outcome, $\bar{y}[\boldsymbol{\beta}_s, \mathbf{x}_s^*]$. When C is in power, group S is surprised by the (wrongly attributed) success of her narrow set of relevant policies, $\bar{y}[\boldsymbol{\beta}, \mathbf{x}_c^*]$, as compared to what she expected, $\bar{y}[\boldsymbol{\beta}_s, \mathbf{x}_c^*]$. Thus the change in the beliefs of S contains a systematic component: A gradual increase in bias and intensity when C is in power and a gradual reduction of the bias when they are in power.

But beyond these two systemic changes in power, another source for power shifts arises from the random shocks ε . Indeed a common thread in the literature on populism is how economic shocks are more likely to lead frustrated voters to support populist movements. We now analyse how the random shocks ε affect power switches. We focus on outcomes close to the steady state: **Proposition 1:** A negative (positive) ε shock to y affects the intensity of the incumbent party relatively more than that of the opposition. Thus a negative (positive) ε shock to y hastens (delays) a regime change.

Random shocks change estimates of the effectiveness of policy, but these effects are stronger for the incumbent party which is implementing its desired policy combination. Specifically, when the simple group is in power, a negative shock reduces their intensity, as their belief in the effectiveness of the policies they deem relevant falls. Complex beliefs in these same policies also fall, but the complex belief in the efficacy of policies the simple deem irrelevant, and hence do not implement, rises, as the poor outcome under simple rule convinces the complex that these neglected policies are more effective than previously thought. These two effects offset each other, and complex intensity remains constant.

When the complex are in power, a negative shock reduces the belief in the effectiveness of policies of both types, but the effects on intensity are greater for the complex, for whom intensity depends upon a wider range of policies, all of which are seen to be failing. In sum, a negative shock hastens the transfer of power, with positive shocks having the opposite effect.

To appreciate these dynamics, we conclude this discussion by simulating Example 1:

Simulation: We now simulate the dynamic process outlined in Example 1, where we describe the mean results for the following parameter values: $\beta_1 = \beta_2 = 1$, R = 1, and the policy shock *n* is normally distributed with $\sigma_n^2 = 0.01$. As a result, $\tau^* = \sqrt{2}$ and thus $\theta_s \approx 0.408$.¹⁹ As for the relative share of time in power, this satisfies $\frac{\theta_c}{\theta_s} \approx 1.45 \approx \tau^*$.

Simulations are run for 10 million periods, and the reported results are for the last 1 million periods. We vary the degrees of the variance of the outcome shock ε , denoted below by σ^2 , increasing it from close to zero to be sufficiently large.

We report in Table 1 on the following long term statistics: First, μ_s denotes the the mean number of periods of simple rule, and μ_c denote the mean number of periods of complex rule. Second, we denote by π_i the fraction of transitions from *i* to *j* which involve a negative outcome shock.

$$\theta_s = \frac{1 - \tau^* \frac{\sigma_n^2}{R}}{1 + \tau^*}.$$

¹⁹Equation (III.13) reports θ_s for the case where $\sigma_n^2 = 0$. The general formulation as we show in the Appendix satisfies:

	$\sigma^2 = .01$	$\sigma^2 = .1$	$\sigma^2 = 1$	$\sigma^2 = 10$	$\sigma^2 = 100$
θ_s	.409	.408	.409	.408	.408
μ_s	1.09	1.41	2.89	7.90	25.5
μ_c	1.58	2.04	4.17	11.48	37.0
$\pi_{ m s}$.526	.662	.908	.987	.999
π_c	.549	.660	.887	.984	.997
$\mu_{cl} \mu_s$	1.45	1.45	1.44	1.45	1.45

Table 1: Simulation of long term transition of power for varying variance of ε .

As can be seen in the table, μ_s and μ_c increase with the variance of ε , where $\mu_c \approx \tau^* \mu_s$. Also, for both types, the fraction of transitions of power π that involve a negative shock increases, from 0.5 to 1. This confirms our analytical results reported in Proposition 1, showing that a negative shock hastens transition of power.

The simulation results illustrate the interplay between the systemic components of power dynamics, which are derived from the equal intensity and colinearity conditions, and those determined directly by the noise ε . The larger is the variance of ε , the more likely are paths in which beliefs wander further away from the point of equal intensity. This lengthens the stay in power of incumbents, as one good shock allows them to be in power for a longer time (noting that future shocks have mean zero). Moreover, the shock is dominating the variation of intensity. Being far away from equal intensity implies that the systematic component cannot easily shift beliefs across the equal intensity point, but a big negative shock will do so.

3.5 Convergence

In general, establishing convergence with misspecified models is problematic even with exogenous iid data (see Berk 1966). Having endogenous data, as we have in our model, introduces more challenges as observations are non iid. As we mentioned in the introduction, substantial progress has been made in the literature analyzing the convergence properties of misspecified models with non iid data.²⁰ But with respect to this literature, our model is further complicated by having multiple players, continuous actions, and a multidimensional state space.

Specifically, multiple dimensions of policy allows for the possibility that types entertain multiple equilibrium beliefs in the long term. This multiplicity introduces additional challenges for establishing convergence as it is hard to prove that types do not perpetually

 $^{^{20}}$ See for example Esponda, Pouzo and Yamamaoto (2019) and Frick, Iijima and Ishii (2020).

"travel" along this continuum of beliefs. As we show below, the policy noise, \mathbf{n} , allows us to establish convergence in this model.

In the appendix we prove convergence with the following steps. First, we establish a law of large numbers for our framework that relies on the fact that at period t, the regressors \mathbf{x}_t and the shock ε_t are independent of each other. While the regressors depend on past realizations of the shock, they are not correlated with the current one. This law of large numbers allows us to show that the beliefs of C converge, with the help of the noise \mathbf{n} , to the true parameters and so $\bar{\boldsymbol{\beta}}_c = \boldsymbol{\beta}$. Given these two steps we can derive a *deterministic* law of motion for the asymptotic beliefs of S.

$$(III.14) \ \bar{\boldsymbol{\beta}}_{s \to}^{a.s} \boldsymbol{\beta}_{s} + c \mathbf{M}^{-1} \boldsymbol{\beta}_{s}, \text{ where}$$
$$\mathbf{M} = \frac{\mathbf{X}_{s}' \mathbf{X}_{s}}{t_{c} R} + \frac{t_{c} + t_{s}}{t_{c}} \frac{\sigma_{n}^{2}}{R} \mathbf{I}_{k_{s}} + \frac{\boldsymbol{\beta}_{s} \boldsymbol{\beta}_{s}'}{\boldsymbol{\beta}' \boldsymbol{\beta}}, \ c = 1 - \frac{\boldsymbol{\beta}_{s}' \boldsymbol{\beta}_{s}}{\boldsymbol{\beta}' \boldsymbol{\beta}}$$

where \mathbf{X}_s denotes the matrix of regressors that S find relevant and have been implemented when S has been in power, and t_i denotes the number of periods type *i* has been in power up to period t (so that $t_i/t = \theta_{it}$).

The policy noise (and sufficient variation of policies along the process) allows S to learn the true relative merits of each policy, which implies that beliefs and policies converge to be colinear. The omitted variable bias captured above by $c\mathbf{M}^{-1}\boldsymbol{\beta}_s$ shifts up and down until asymptotic power sharing results in just enough bias to reach equal intensity, and the dynamics of this are similar to those described in the one-dimensional case.²¹

4 Extensions

In this Section we present some additional results and discuss alternative modelling assumptions.

4.1 Overlapping versus non-overlapping world views

Wrong complex world view. Above we considered an environment in which the beliefs of the complex are correctly specified, in that they include all relevant policies, whereas the simple type erroneously exclude a subset of these. The fact that the complex consider all relevant policies does not matter at all, and it is sufficient for our results that the simple consider a subset of the relevant policies that the complex consider.

Overlapping world views. One can consider the model in which the two groups have overlap in the policies that both consider relevant, but that each group considers in addition

²¹The Appendix illustrates the phase diagrams derived from (III.16).

an exclusive set of additional relevant policies. In the crime prevention example one group can consider police funding and border controls, whereas the other can consider police funding as well as employment opportunities and integration policies. We can show that if beliefs converge, there is always a steady state equilibrium in which one of the groups "becomes" a simple group in the sense that it abandons its exclusive relevant dimensions. In this steady state our results about political cycles as well as the extremism of the simple group's policies still hold. However, we do not have convergence results for this type of environment.

Non-overlapping world views. Another possibility is that there is no overlap between the world views of the groups, so that each group considers a mutually exclusive set of relevant policies. In these cases we can have one group perpetually in power. For example, if one group has a low prior on how its policies affect output, then the other group can potentially be in power in the long term.

Irrelevant policies. In Appendix I we consider an additional extension in which the relation between the two groups' world views is the same (simple versus complex) when it comes to relevant policies but both types can also consider policies which are irrelevant. We show that all the results reported in the paper hold for this more general model. Our assumption of noise implies that both groups abandon the non-relevant variables in the long term and hence the asymptotic equilibrium looks exactly like the one in our basic model.

4.2 Strategic Politicians

We use a simple political model in which intensity of preferences is the key to electoral success. We adopt a citizen candidate model so that politicians choose policies myopically, and offer voters exactly their ideal polices. There are many reasons to justify the assumption of myopia in politics as politicians have limited terms, policy choices have high stakes and it is very complicated to predict the influence of current policies on future behaviors. We now discuss two alternative assumptions.

Strategically affecting the beliefs of the other group: One element of our model is that we assume that the winning politician implements her myopic ideal policy. That is: (i) she does not engage in experimentation in order to enhance her learning; (ii) she does not use today's policy choice to manipulate future learning and actions of others.

With regards to (i), our assumption of policy implementation noise captures some form of experimentation. Indeed, this feature of the model is the reason why the complex end up converging on the true parameters of their model. Below, when we consider the steady state equilibria without such noise, additional equilibria arise in which the complex do not learn the true parameters of their model.

More sophisticated forward strategic behavior along the lines of (ii) might alter some of

our positive results but not the qualitative effect of the simple group's influence on policy outcomes. While this is beyond the scope of our analysis, we conjecture that even in such a model, in the long term, the simple group's misspecified model will affect policies. Specifically, it is not possible for the complex group to be perpetually in power implementing their ideal policy, as in such a case the simple's estimates must converge to induce them to have higher intensity. As a result, even if the complex converge to be in power perpetually, they must implement long-term policies that prevent the simple from obtaining higher intensity; such policies have therefore to be biased.

Office-motivated politicians: One may imagine other models of political competition, e.g., a probabilistic voting model with office motivated politicians, which essentially implies that politicians choose policies to maximize average welfare. While this would yield different policies as well as learning patterns, a key feature of our analysis will remain: In equilibria, policies will cater to group S to some degree. That is, the omitted variable bias in S's beliefs would mean that they would prefer stronger policies on the policies they deem effective. Any policy that maximizes welfare will then exhibit such a bias.

4.3 Endogenous resources and other utility functions

In our model we have assumed a fixed resource constraint R. We can extend the model to allow the different types to endogenously choose their desired level of resources. In particular, we can assume that the utility function of citizens is given by:

$$U_t = y_t + V(R_t),$$

where as before $R_t = \mathbf{x}'_t \mathbf{x}_t$ represents the resources used in implementing policy \mathbf{x}_t for y_t , while V represents the utility derived from policy outcomes over which there is no disagreement regarding causal mechanisms. V is a reduced form, representing the utility that can be achieved in other policy areas given the allocation of resources to y_t , and the assumptions V' < 0 and V'' < 0 are natural. To derive analytical results, we work with a second-order approximation of V as a quadratic function of R_t . We can then show that intensity of preferences is also an increasing function of the magnitude of beliefs. Assuming that R_t is bounded from above, we can then extend all our convergence results accordingly.

We also assume a simple utility function that is linear in y, which implies that utility is a function of mean beliefs only. For more general utilities the whole distribution of beliefs would matter. Montiel Olea et al (2019) show that in a model with exogenous data, complex models (which abide with the truth) would induce lower variance of their beliefs when data is sufficiently large. This would imply an advantage to the complex group. Thus, our results hold as long as individuals are not too risk averse.

4.4 Relation to Berk-Nash equilibrium

To conclude the discussion, we examine the relation between our results above and a static notion of equilibrium in the spirit of Berk-Nash equilibrium (Esponda and Pouzo 2016). A Berk-Nash equilibrium is a static solution concept for a dynamic game of players with misspecified models where actions are optimal given beliefs and beliefs rationalize the observed output which arises given the actions played. Berk (1966) shows for the case of iid data that beliefs stemming from a misspecified model will concentrate on those that minimize the Kullback–Leibler (KL) distance to the true beliefs.²² Using this notion of minimizing the KL distance, Esponda and Pouzo (2016) define a Berk-Nash equilibrium.

In the Appendix we analyse the set of Berk-Nash equilibria in our model, where we assume away the policy noise and focus on a more general Bayesian framework. Analogously to our dynamic cycles, in the static solution, $\theta_s \in [0, 1]$ which denotes the probability that type S is in power. We show then using similar intuition to the one provided in Section 3.1, that our equilibrium identified in Theorem 1 also constitutes a Berk-Nash equilibrium (Proposition A1 in Appendix III). However, there can be other Berk-Nash equilibria in our model. Still, Proposition A2 shows that policy inefficiency is an inherent feature of any Berk-Nash equilibrium. Specifically, we show that any equilibrium will be characterized either by $\theta_s > 0$ or by group C having zero expected beliefs on some of its relevant policies.

5 Conclusion

Our analysis has shown how simplistic beliefs can persist in political competition against a more accurate and complex view of the world, delivering sub-par outcomes on each outing in power and yet returning to dominate the political landscape over and over again. In the framework presented above simplistic beliefs arise as a consequence of a primitive assumption of misspecification, but we recognize that there are deeper questions to explore. A recent examination of European Social Survey data by Guiso et al (2017) finds that the responsiveness of the electorate to populist ideas and the supply of populist politicians increases in periods of economic insecurity. Social and economic transformation, and the insecurity and inequality it can engender, may create environments in which opportunistic politicians are able to plant erroneously simplistic world views into the electorate. Linking belief formation, at its most fundamental level, to ongoing economic and political events allows a richer characterization of political cycles, and is something we intend to explore in future work.

²²Intuitively, minimising the Kullback–Leibler distance is similar to maximising the likelihood of previous observations.