Predicting Time to Upgrade for Successive Product Generations:
An Exponential-Decay Proportional Hazard Model

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Abstract

In the presence of successive product generations, most consumers are repeat buyers, who may decide to purchase a future product generation before its release. As a result, after the new product generation enters the market, its sales often exhibit a declining pattern, thus rendering traditional bell-shaped product life-cycle models unsuitable for characterizing consumers’ time to product upgrades. In this study, we propose an Exponential-Decay Proportional Hazard Model (Expo-Decay model) to predict consumers’ time to product upgrade. The Expo-Decay model is parsimonious and easy to interpret and performs better than or as well as existing models in prediction accuracy. We apply the Expo-Decay model as well as three extensions to study consumers’ upgrade behaviors for a sports video game series. Empirical results reveal that consumers’ previous adoption and usage patterns can help predict their timing to product upgrades. In particular, we find that (i) consumers who have adopted the immediate past product generation are more likely to upgrade; (ii) players who play previous generations more often tend to upgrade earlier; (iii) consumers who specialize in a small subset of game modes demonstrate a lower probability to upgrade. When comparing the Expo-Decay model and its extensions, we find that more complex model extensions do not lead to better prediction performance than the baseline Expo-Decay model, while a time-variant extension that updates the values of covariates over time outperforms the baseline Expo-Decay model with static data.

Keywords: Predictive analytics, product upgrade, survival analysis, proportional hazard model
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1. Introduction

Continuous product improvement and frequent releases of new product generations are a common practice by firms. Releasing improved product generations enhances consumer loyalty and encourages more repeat purchases (Albuquerque and Nevskaya 2012), thereby sustaining or increasing sales that otherwise would decline as a result of market saturation. For example, Call of Duty, the best-selling first-person shooter video game series, releases new game generations every year to keep players engaged and safeguard its market dominance. Since the first introduction of iPhone in 2007, a new generation of iPhone has been released about once every year, mostly in the summer months, from 2008 to 2020. Similarly, Samsung introduced its first generation of Galaxy phones in 2009, and since then has been releasing multiple generations/models to the market every year in the last 10 years.¹ In fact, continuous product improvement is so common that it is now difficult to find firms that do not release new product generations from time to time.

As a product line matures and gains consumer loyalty, the market response to a new product generation typically differs substantially from that to the first product generation. Specifically, since the uncertainty about the quality and functionality of a new generation all but disappears, returning consumers no longer have the “wait and see” attitude toward a new generation. As a result, we often observe a sales growth curve that is drastically different from that of the first product generation—the rate of sales is at its highest level upon the release of a new generation, then gradually fades over time. For instance, on average, 17% of iPhone users upgrade as soon as a new model is released, 58% upgrade one year after the release, and 22% two years after the release; only 2% of users wait longer (Edwards 2016).

That being said, new product generations may not always be popular. If the quality improvement is marginal, consumers may be reluctant to upgrade to a new generation. For instance, in recent years, the average time-to-upgrade for smartphones has increased. In 2014, U.S. consumers are upgrading their smartphones every 23 months. Lately, consumers on average are holding onto their phones for eight more months. It is estimated that the time gap between upgrades would widen further in the coming years (Martin and FitzGerald 2018). Therefore, it is important to identify factors that might reveal existing users’ upgrade intentions.

A limited number of prior studies have focused on factors that might impact consumers’ future purchases intentions for a new product series. To the best of our knowledge, no prior research has examined the association between consumers’ previous adoption and usage experience and their upgrade timing decisions. Furthermore, in the presence of successive product generations, most consumers are repeat buyers, who may decide to purchase a future product generation before its release. As a result, after the new product generation enters the market, its sales often exhibit a declining pattern, thus rendering traditional bell-shaped product life-cycle models unsuitable for characterizing consumers’ time to product upgrades. Motivated by this declining sales trend, the present study proposes an Exponential-Decay proportional hazard model (Expo-Decay model for short) to help explain and predict consumers’ upgrade behaviors.

Using a rich dataset for a sports video game series that includes individual-level activation and usage records, we evaluate the proposed Expo-Decay model against existing survival models, and identify the key predictors of time-to-upgrade decisions. Our test shows that the Expo-Decay model performs better than or as well as existing models in prediction accuracy. Empirical results also confirm that players’ prior adoption and usage experience can indeed help predict their timing of product upgrade. In particular, we find that consumers who have adopted the immediate past generation are more likely to upgrade to a new generation, players who play previous generations more often tend to upgrade earlier, and consumers who specialize in a small subset of game modes demonstrate a lower probability to upgrade.
In addition to the baseline Expo-Decay model, we develop and test three extensions of the model: (i) a frailty model extension that incorporates unobservable consumer heterogeneity, (ii) a double Expo-Decay extension that captures the influences of previous adoptions, and (iii) a time-variant extension that updates the values of covariates as time progresses. Further empirical tests using the video game dataset reveal that more complex model extensions do not lead to better prediction performance than the baseline Expo-Decay model, while a time-variant extension that updates the values of covariates over time outperforms the baseline Expo-Decay model with static data.

In the next section, we review related prior literature, particularly on factors that drive consumers’ upgrade decisions. In Section 3, we propose the Expo-Decay model. Section 4 describes our video game dataset, experience-based covariates, and some model-free evidence. Empirical estimations and findings are presented in Section 5. In Section 6, we propose and test three model extensions. We conclude the paper in Section 7 with discussions on main contributions, managerial implications, and future research directions.

2. Related Literature

In this section, we review three streams of literature that are relevant to the present study, on (a) incentives of product adoptions and upgrades, (b) influence of consumers’ previous experience on product upgrade, and (c) duration models for product upgrade, respectively.

2.1. Incentives of Product Upgrade

In the diffusion of innovations literature, researchers have attempted to identify the influence of consumer characteristics on product upgrades. For instance, potential adopters’ income level, education, occupation, and experience with other related technical products are found to influence their upgrade propensity toward a new technology (Dickerson and Gentry, 1983). Psychologically, venturesome, impulsive, flexible, and inner-directed innovators are expected to be more open to technology upgrades (Huh and Kim 2008).
In a multigeneration product series, characteristics of a new generation often create need arousal for upgrades. Van Nes and Cramer (2008) find that product characteristics, including technological performance, hedonic value, features and technological advantages, psychological value, ergonomics, economic value, and ecological benefit, may motivate consumers to upgrade to a new product generation. In addition, several moderators, such as promotional formats, usage frequency (Okada 2001), product similarity (Okada 2006), trade-in conditions (Purohit 1995), and transaction conditions (Zhu et al. 2008), may influence consumers’ upgrades decisions.

It is important to note that, in identifying factors that drive consumers’ upgrade decisions, prior studies in this research stream have focused on consumers’ perception of the features of a new product (generation) and a firm’s marketing effort, while ignoring consumers’ purchases and usages of previous product generations. This study fills this void and utilizes consumers’ past experience to predict their future upgrade decisions.

2.2. Consumer Experience and Upgrade Decisions

Aside from characteristics of technology improvements and consumers’ demographic and psychographic factors, consumers’ experience with related technologies are found to impact their upgrade decisions (Dee Dickerson and Gentry 1983). Based on factors identified in the prior literature, Kim et al. (2001) find that previous adoption history and post-adoption behavior toward current products are more robust predictors of upgrade decisions. Similarly, Shih and Venkatesh (2004) point out two perspectives for innovation diffusion studies: an adoption-diffusion perspective, which examines the process through which a target population adopts an innovation, and a usage-diffusion perspective, which aims to identify the usage behavior associated with an innovative product. Adopting the two perspectives, we examine how consumers’ past adoption and usage experience can help predict their time-to-upgrade decisions.

With the help of modern data collection and storage technologies, consumers’ previous adoption experience has become more available than ever before. Rijnsoever and Oppewal (2012) show that variables associated with previous adoptions outperform conventional socio-demographic and
psychographic variables in helping predict early adoptions. Prior studies have shown that successful adoption experience with previous product generations may improve the expected benefits of an entire product series, therefore reducing resistance against similar technologies (Shih and Venkatesh 2004, Chang et al. 2005). Conversely, researchers have also found that prior adoption experience could also hinder the adoption of a new generation. For instance, some studies find that a consumer’s motivation to upgrade likely decreases if the version she has already adopted can fulfill her needs (Ellen et al. 1991, Gerlach et al. 2014). Further, consumers who like certain attributes of existing products might even negatively react to a substitute that differs on those attributes (Ellen et al. 1991).

In addition to previous adoption experience, consumers’ previous usage experience can help gain insight into their upgrade decisions. Sääksjärvi and Lampinen (2005) first study how usage experience with a previous product generation plays a role in affecting the perceived risk of adopting a successive generation. Shih and Venkatesh (2004) conceptualize innovation usage with two distinct dimensions, variety of use and rate of use, resulting in four distinct usage patterns: intense, specialized, nonspecialized, and limited. They suggest that users demonstrating a higher usage patterns are more open to future technologies compared to users exhibiting a lower usage pattern.

Despite the rich literature in this space, there is a clear need for better understanding of the influence of prior product usage experience on future upgrade decisions. We believe that this void is at least partially attributed to a lack of relevant data—consumers’ product usage records are typically not observable to analysts or researchers. Armed with a rich dataset that records adoptions as well as usages of a sports video game series, the present research aims to analyze consumers’ product adoption and usage behaviors and how such behaviors can help predict their product upgrade decisions.

2.3. Time-to-Upgrade Models
Some prior studies have attempted to model the timing of repeat purchases. Drawing on the stochastic counting and timing literature, researchers have developed models of repeat purchase behaviors that utilize data from firms’ transaction databases for model training and development. In particular, the
negative binomial distribution (NBD) model is found to provide an excellent fit to repeat purchase data (Ehrenberg et al. 2004), which assumes that the number of purchases made by a consumer in a given time period can be characterized by a Poisson distribution with the buying rate following a gamma distribution.

Only a few of the prior studies have modeled the purchase timing of high-tech product upgrades, among which the survival model, specifically the proportional hazard model, is the most widely applied. Kim and Srinivasan (2009) propose a conjoint utility model with a hazard function specification to examine the upgrade timing of PDAs. Extending the conventional duration model, Sinha and Chandrashekaran (1992) first develop a split hazard model to analyze the diffusion of innovations, in which the splitting model indicates whether a consumer will eventually adopt the product while the hazard part models the distribution of the time to adoption. Prins and Verhoef (2007) apply the split hazard model to study the effect of marketing communication on existing consumers’ adoption timing of a new E-service.

In this study, following the survival analysis framework, we develop an efficient and parsimonious proportional hazard model to study how existing consumers’ adoptions and usage experience can help predict their timing of product upgrades. Based on the baseline model, we also propose and test several extension models that capture unobservable consumer heterogeneity, consumers’ complete adoption history, and time-variant covariates.

3. Time-to-Upgrade Model Development

Firms often release successive product generations based on a predetermined schedule. For instance, new generations of the video game Call of Duty are usually released in late October or early November every year. The present study focuses on this type of product series, for which the product series have accumulated some consumer base and the release dates of past and future generations are considered public knowledge.

As illustrated in Figure 1, at around the same time each year, the focal firm releases an improved generation of its product series. Consumers may or may not upgrade to the newest generation every year.
For example, the consumer illustrated in Figure 1 adopted the first generation (G1) \( t_1 \) days after its release, but did not upgrade after the second generation (G2) became available. Now, after the third generation (G3) is launched, the consumer decides whether to upgrade to G3, or continue to use G1 and wait for a further improved future generation.

![Figure 1. An Illustrative Example of Cross-Generation Adoptions](image)

Based on the theoretical discussions in the previous section, the time to upgrade to a new generation depends on many drivers. In this study, we focus on factors that reflect consumers’ previous adoption behaviors and usage patterns. In this section, we briefly review the survival analysis method and then propose a new proportional hazard model to examine how consumers’ past experience can be used to understand and predict their timing of upgrade purchases.

### 3.1. Proportional Hazard Model

It is important to note that common prediction models in machine learning, such as classifiers (e.g., logistic regression) and numerical prediction models (e.g., linear regression) are not suitable for predicting the time to an event. An appropriate choice for our task is survival models, which can predict the probability density of time to event, and can effectively address the issue of right-censored data (Helsen and Schmittlein 1993).
Time-to-event survival analysis has been widely applied in business research. For instance, researchers have employed survival models to study the time duration between consumers’ repeat purchases (Jain and Vilcassim 1991, Seetharaman and Chintagunta 2003), and to predict the propensity, frequency, and timing of readmissions of patients (Bardhan et al. 2014).

In this study, we apply a proportional hazard model (PHM) specification (Cox 1972) to explain and predict consumers’ time-to-upgrade decisions, because PHMs can capture the influence of covariates of interest and provide better interpretability than alternatives. The baseline hazard rate, defined as the (instantaneous) probability of upgrade during an infinitely small time-interval \((t, t + \Delta t)\) conditional on no upgrade having occurred before time \(t\), is

\[
h(t) = \lim_{\Delta t \to 0} \frac{p(t < T < t + \Delta t | T > t)}{\Delta t} = \frac{f(t)}{S(t)} = \frac{f(t)}{1 - F(t)},
\]

where \(f(t)\) and \(F(t)\) are the probability density function and cumulative distribution function of the upgrade timing, respectively. The survival function \(S(t)\) represents the probability that there is no upgrade before time \(t\). A survival process can be characterized by the hazard function, the probability density function, or the survival function.

The covariates of interest can be incorporated into the baseline hazard rate to generate the consumer-specific hazard rate:

\[
h(t, X_i) = h_0(t) e^{X_i' \beta},
\]

in which \(h_0(t)\) is the baseline upgrade hazard, \(X_i\) is a vector of covariates representing consumer \(i\)’s previous adoption and usage experience, and the coefficient vector \(\beta\) captures the relationship between these covariates and the time-to-upgrade. Incorporating these experience-related covariates, the survival function becomes

\[
S(t, X_i) = [S_0(t)]^{\exp(X_i' \beta)},
\]

in which the baseline survival function, \(S_0(t)\), can be derived from the baseline hazard function \(S_0(t) = e^{- \int_0^t h_0(u) \, du}\). The probability density function of time-to-upgrade takes the form:

\[
f(t, X_i) = h_0(t) e^{X_i' \beta} [S_0(t)]^{\exp(X_i' \beta)}.
\]
### 3.2. Common Baseline Hazard Functions

In a PHM, \( h_0(t) \) describes how the upgrade hazard rate changes over time in the absence of influences from related covariates. It is not always necessary to explicitly specify a baseline hazard function in survival analysis. For instance, a well-known semi-parameter model, the Cox model, does not need a baseline hazard function.

#### Table 1. Parametric Baseline Hazard Functions

<table>
<thead>
<tr>
<th></th>
<th>Exponential</th>
<th>Weibull</th>
<th>Erlang-2</th>
<th>Box-Cox</th>
<th>Expo-Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_0(t) )</td>
<td>( \gamma )</td>
<td>( \gamma \alpha (\gamma t)^{\alpha - 1} )</td>
<td>( \frac{\gamma^2 t}{1 + \gamma t} )</td>
<td>( \exp \left[ \gamma_0 + \sum_{k=1}^{K} \gamma_k \left( \frac{t^{\lambda_k} - 1}{\lambda_k} \right) \right] )</td>
<td>( \gamma \alpha t^{\alpha - 1} e^{\theta \alpha} )</td>
</tr>
<tr>
<td>( S_0(t) )</td>
<td>( e^{-\gamma t} )</td>
<td>( e^{-(\gamma t)\alpha} )</td>
<td>( (1 + \gamma t)e^{-\gamma t} )</td>
<td>--</td>
<td>( \frac{\gamma}{e^{\beta [1 - e^{\theta \alpha}]} } )</td>
</tr>
<tr>
<td>Shape of Baseline Hazard</td>
<td>Flat, monotonically increasing, monotonically decreasing</td>
<td>Flat, monotonically increasing</td>
<td>Monotonically increasing</td>
<td>Flat, monotonically increasing, monotonically decreasing, U-shaped, or inverted U-shaped</td>
<td>Flat, monotonically increasing, monotonically decreasing, U-shaped, or inverted U-shaped</td>
</tr>
</tbody>
</table>

For parametric PHMs, there are a few widely applied baseline hazard specifications, including Exponential, Weibull, Erlang-2, and Expo-Power. Their underlying hazard functions and survival functions are summarized in Table 1. The exponential hazard assumes a constant hazard rate, which is a special case of the Weibull hazard. The Weibull hazard can capture constant, monotonically increasing, and monotonically decreasing hazard rates, and is the most widely applied in the PHM literature. The Erlang-2 hazard has a monotonically increasing shape and has been used in estimating consumers’ inter-purchase time distributions (e.g. Seetharaman 2004). In a comparison study, Seetharaman and Chintagunta (2003) review different specifications of PHM and find that the flexible Expo-Power specification fits data the best. In another study, Jain and Vilcassim (1991) show that most probability distributions suggested in marketing literature can be nested within the Box-Cox formulation. In this study, the widely applied Weibull model, the most flexible Expo-Power model, as well as the Box-Cox...
model are selected as benchmarks. With respect to the Box-Cox formulation, we apply a restricted version of the general model in empirical validation, in which \( K = 3 \), and the baseline function is

\[
h_0(t) = \exp \left[ \gamma_0 + \gamma_1 \left( \frac{t^{\lambda_1} - 1}{\lambda_1} \right) + \gamma_2 \left( \frac{t^{\lambda_2} - 1}{\lambda_2} \right) + \gamma_3 \left( \frac{t^{\lambda_3} - 1}{\lambda_3} \right) \right].
\]

\( \gamma_0, \gamma_k, \lambda_k \) \( (k \in \{1, 2, 3\}) \) are parameters to be estimated and \( t \) is the time to upgrade. Following Jain and Vilcassim (1991), in this restricted model, we fix \( \lambda_1 = 1 \), \( \lambda_2 \to 0 \), and \( \lambda_3 = 2 \). The baseline function that can be used as the general Box-Cox in estimation thus reduces to:

\[
h_0(t) = \exp \left[ \gamma_2 \ln(t) \right] \cdot \exp \left[ \frac{\gamma_2}{2} t^2 + \gamma_1 t + \gamma_0 - \gamma_1 - \frac{\gamma_3}{2} \right]. \tag{4}
\]

3.3. Pre-release Adoption

Prior to the release of a new product generation, companies often advertise it through various channels. For instance, official trailers of a new video game generation may be posted on Youtube.com months before its release date, and short demo-version games can be made available for download on platforms a few weeks before the game launch date. More importantly, existing consumers, which often account for the majority of adopters of a new generation, could be aware of an upcoming new generation even if they are not exposed to such advertisements. As a result, most potential consumers of an upcoming product generation may be well informed of the time frame of the release of the new generation, and have reasonable prior expectations about the quality of new product generation.

Due to the pre-release product awareness and diffusion of information, potential consumers who anticipate a forthcoming new product generation may commit to buy it prior to its release. In fact, empirical evidence shows that the pre-release word-of-mouth (WOM) dynamics can serve as early indicators of future product sales, and products with more pre-release WOM tend to have higher initial sales (Gelper et al. 2014). Hence, consumers’ upgrade decisions might have been made before the release of the new product generation, which we refer to as virtual adoptions in the rest of the discussion.

Despite the pre-release virtual adoptions, actual sales or activations can only take place after the release of a new product generation. Upon its release, most accumulated virtual adopters would purchase the new
product generation they’ve been waiting for in a short time-frame following the product launch, as evidenced by the scene of long waiting lines following the release of a new iPhone generation. As a result, the hazard rate is the highest at the very beginning of the product sales window. As more and more virtual adopters as well as enthusiasts have made their upgrade purchases, the (instantaneous) probability of upgrading by the remaining consumers tends to decrease over time, leading to a declining hazard rate. This declining hazard rate renders the traditional product lifecycle models and the associated bell-shaped growth curve unsuitable for modeling the upgrade sales of a new product generation. To address this problem, the present research proposes a parsimonious and flexible baseline hazard function to capture the declining upgrade hazard rate of a new product generation.

3.4. The Exponential-Decay Proportional Hazard Model

Based on our extensive literature review, most existing diffusion models and time-to-purchase models proposed in the prior literature cannot effectively capture a declining hazard trend. For instance, the hazard function of the classic Bass model (Bass 1969) is monotonically increasing with time. The parsimonious BOXMOD-I framework proposed by Sawhney and Eliashberg (1996), which is a generalization of Exponential, Erlang-2, and Generalized-Gamma distributions, characterizes a non-decreasing baseline hazard function. Therefore, it is imperative to develop a model that could effectively capture the declining hazard trend. In this study, we propose a parsimonious baseline hazard function, which we name as Exponential Decay (Expo-Decay) baseline hazard function, as follows:

\[ h_0(t) = \gamma * e^{-\alpha t}, \alpha, \gamma > 0. \] (5)

The associated survival function is

\[ S_0(t) = e^{(\frac{T}{\alpha}) + (e^{-\alpha t} - 1)}. \] (6)

We refer to \( \gamma \) in the Expo-Decay function as the scale parameter, and \( \alpha \) as the decay rate parameter. Although theoretically the Expo-Decay function can capture flat, increasing, or decreasing hazard rates, we are only interested in the declining curve it provides, hence we have \( \alpha, \gamma > 0. \) In the remaining
discussion, we refer to this survival model as the *Exponential Decay Proportional Hazard model* or simply the *Expo-Decay model*.

Compared with alternative baseline hazard functions, the Expo-Decay model has two important advantages. First, the Expo-Decay function requires only two parameters, and has as parsimonious a form as the widely applied *Weibull* function, and simpler than the Expo-Power function and the Box-Cox formulation. Second, the Expo-Decay has better interpretability than other specifications. Specifically, the scale parameter ($\gamma$) helps capture the magnitude of willingness to upgrade right after the release of the new generation, and the decay rate parameter ($\alpha$) reflects how quickly the hazard rate declines over time. Third, the parsimonious and interpretable form of the Expo-Decay model makes it more likely to be adopted and extended in future empirical and analytical research, and be applied in practice. The advantages of a simple and interpretable model can hardly be overstated and is well documented in related disciplines.\(^2\)

Although the hazard rate $h(t,X_i)$ is a continuous-time function, observable data is typically in discrete form. Therefore, based on the continuous model, we define a discrete upgrade hazard rate for consumer $i$ during time interval $j$, $(t_{j-1},t_j]$ as

$$
\lambda(j,X_i) = Prob(T_i \leq t_j|T_i > t_{j-1}) = 1 - e^{-[H(t_j,X_i)-H(t_{j-1},X_i)]},
$$

(7)

where $H(t_j,X_i)$ is the cumulative hazard function, i.e., $H(t_j,X_i) = \int_0^t h(u,X_i)\,du$. The discrete hazard rate $\lambda(j,X_i)$ represents the probability that consumer $i$ will upgrade during the $j$th interval given she has not upgraded before $t_{j-1}$. The discrete survival function for time interval $j$, $(t_{j-1},t_j]$, is:

$$
S(t_j|X_i) = \exp\left\{-\int_0^{t_j} h(\tau|X_i)\,d\tau\right\} = e^{-H(t_j,X_i)}.
$$

(8)

In general, if consumer $i$ does not upgrade in the observation window of a dataset (right censored), the likelihood is the probability of survival till the end of the observation window: $L_i = S(t_D,X_i)$, where $t_D$ denotes the last observed time interval. When consumer $i$ upgrades during time interval $t_i$, the

\(^2\) Take the product diffusion models in marketing as an example—although there are numerous extensions to the seminal Bass (1969) Model, the simpler Bass Model remains the most well-known and most cited diffusion model to date (Rogers 2003).
likelihood is the probability that consumer $i$ has survived till the end of time interval $\tau_{i-1}$ multiplied by the upgrade hazard rate during $(\tau_{i-1}, \tau_i]$, i.e., $L_i = S(\tau_{i-1}, X_i) \cdot \lambda(\tau_i, X_i)$. Therefore, the joint likelihood function for all consumers in a data sample is

$$L = \prod_{i=1}^{N} L_i = \prod_{i=1}^{N} \left[ S(\tau_{i-1}, X_i) \cdot \lambda(\tau_i, X_i) \right]^{\delta_i} \left[ S(t_D, X_i) \right]^{1-\delta_i}, \quad (9)$$

where $\delta_i$ is a binary indicator of the upgrade status of consumer $i$. From Eq. (9), we can use the Maximum Likelihood Estimation (MLE) method to estimate the model parameters.

Since the present study examines the upgrade decisions of existing consumers in a setting influenced by pre-release virtual adoptions, we only compare the Expo-Decay model with ones that can capture a declining hazard trend, i.e. Weibull, Box-Cox, and Expo-Power specifications, in our performance evaluations. Details about the empirical analysis are provided in Section 5.

4. Data Description

We apply the Expo-Decay model to study consumers’ upgrade behaviors for a major sports video game series produced in North America. The game is mainly played on gaming consoles such as PlayStation and Xbox. The publisher of the game releases a new generation in the same month every year. For simplicity, the game generation is labeled based on its release year. For instance, the generation released in 2011 is labeled as G-11.

4.1. Data Sample

The dataset we use includes transactional records of product activations, which is used to approximate sales, game playing sessions, and in-game purchases by game console players, which enable us to investigate consumers’ product adoption, usage, and upgrading behaviors from multiple perspectives.

Players’ activations are recorded for generations G-10 through G-16. The video game can be purchased from a brick-and-mortar store or online through a game console. For each generation, ten thousand unique players are sampled based on activation records, resulting in more than 60,000 unique players being tracked across multiple generations of the game series. However, game playing session
records are only available for G-12 through G-16, and in-game enhancement purchases are collected for G-11 through G-16. To examine the impacts of previous experience (i.e. adoption and usage) on players’ future upgrades, we can only utilize playing session records and in-game purchases data starting from G-12 to explain upgrades starting from G-13. In the empirical analysis, we focus on upgrade purchases of G-15; therefore, a sample of 34,584 unique players who have active usage and activation records before the release of G-15 are selected in the data sample we use for empirical testing.

4.2. Variable Description/Measures
To understand how consumers’ experience impacts their upgrade decisions, we summarize consumers’ past adoption and usage experience by extracting related covariates from transaction records. The summary statistics of these covariates are provided in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Label</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Past Adoption Experience</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of generations activated by the consumer</td>
<td>NumGens</td>
<td>1.51</td>
<td>0.74</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Time interval between release dates and activation dates for adopted generations</td>
<td>WaitDays</td>
<td>167.55</td>
<td>191.38</td>
<td>-11</td>
<td>1096</td>
</tr>
<tr>
<td>Whether the consumer has activated the immediate past generation</td>
<td>Switch</td>
<td>0.51</td>
<td>0.50</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Past Usage Experience</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of game sessions the consumer has played</td>
<td>NumSess</td>
<td>36.90</td>
<td>68.60</td>
<td>1</td>
<td>1920</td>
</tr>
<tr>
<td>Number of game modes the consumer has played</td>
<td>NumModes</td>
<td>4.49</td>
<td>3.47</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>Gini coefficient of the allocation of time spent on different game modes</td>
<td>GiniIndex</td>
<td>0.90</td>
<td>0.06</td>
<td>0.56</td>
<td>0.97</td>
</tr>
<tr>
<td>Number of in-games enhancement purchases a consumer has made</td>
<td>EnhancePurchase</td>
<td>2.12</td>
<td>19.47</td>
<td>0</td>
<td>1494</td>
</tr>
<tr>
<td>Time interval between a consumer’s latest game session and the release date</td>
<td>RecentActDay</td>
<td>304.08</td>
<td>282.56</td>
<td>0</td>
<td>1019</td>
</tr>
</tbody>
</table>

Since early adopters of previous generations tend to upgrade earlier (Huh and Kim 2008), we use \( \text{WaitDays} \) to denote how long a consumer waited after product release to activate the game generation she
is currently using. Loyal consumers are usually more willing to make future purchases, so we count the number of product generations \((\text{NumGens})\) a consumer has previously adopted. The dummy variable \(\text{Switch}\) indicates whether a consumer is a potential \textit{switcher}—those who have adopted the immediate past generation in the game series, or a potential \textit{leapfrogger}—those who have adopted one or more earlier generations but not the most recent one.

Players’ game usage experience is summarized following the \textit{rate of use} and \textit{variety of use} perspectives. Specifically, the number of game sessions a player has played \((\text{NumSess})\) is counted to measure the rate of use. Two variables are generated to capture the variety of use: \(\text{NumModes}\), the number of game modes a consumer has played, and \(\text{GiniIndex}\), the Gini coefficient of the allocation of time among different game modes. Players with high \(\text{GiniIndex}\) values spend most of the gaming time on a limited number of game modes. The \(\text{GiniIndex}\) approaches 0 when the player allocates time evenly across all game modes. Other usage related variables, such as \(\text{EnhancePurchase}\) and \(\text{RecentActDay}\), are also included. \(\text{EnhancePurchase}\) counts how many in-game enhancement purchases a player has made in one game generation. Since enhancement packs purchased in one generation cannot be applied in other game generations, enhancement purchases may imply sunk costs and switching costs, hence impacting players’ upgrade decisions. \(\text{RecentActDay}\) is defined as the time interval between a consumer’s latest game session date and the release date of a new generation. Recently active players are expected to have a fresh memory about the game features and show higher willingness to upgrade.

In general, existing players demonstrating active usage patterns are expected to be more open to future technologies. In other words, a consumer who has started a larger number of game sessions, played more game modes, made more enhancement purchases, and played the game more recently is expected to demonstrate a higher probability of upgrading.

To rule out the multicollinearity concerns, the \textit{variance inflation factor} \((\text{VIF})\) (Kutner et al. 2004) analysis is conducted for these intrinsic experience-based variables. The results in Table 3 show all VIF values are below 10, and the high value for \(\text{NumModes}\) reflects its high correlation with \(\text{GiniIndex}\).
Therefore, we remove \textit{NumModes} from our empirical model; all VIF values become quite low after the deletion, thus eliminating potential concerns regarding multicollinearity.

\begin{table}[h]
\centering
\caption{VIF Values for Multicollinearity Check}
\begin{tabular}{|l|c|c|}
\hline
\textbf{Variables} & \textbf{VIF} & \textbf{VIF Revised} \\
\hline
\textit{NumGens} & 1.48 & 1.47 \\
\textit{WaitDays} & 1.97 & 1.72 \\
\textit{Switch} & 2.76 & 2.66 \\
\textit{NumSess} & 2.29 & 1.48 \\
\textit{NumModes} & 7.83 & --- \\
\textit{GiniIndex} & 4.61 & 1.12 \\
\textit{EnhancePurchase} & 1.10 & 1.10 \\
\textit{RecentActDay} & 3.02 & 2.54 \\
\hline
\end{tabular}
\end{table}

4.3. Model-Free Evidence

Figure 2 shows how the composition of sales for the video game series changes from generation to generation during the data observation window. In this figure, \textit{n-G} leapfrog captures players who skipped \textit{n} generations in the middle before activating the focal generation. Due to data truncation problem, the \textit{new and others} in Figure 2 include new adopters and leapfroggers who have skipped more generations than we could track. The 1-G and 2-G leapfrogs can be identified but they only account for a small proportion (around 7% to 10%) of sales. It is worth noting that among all identifiable consumers, \textit{switching players}, those who upgraded from the immediate past generation, account for the largest proportion of purchases.

![Figure 2. Composition of Sales for Each Game Generation](image-url)
Before specifying any baseline hazard function, we apply the Kaplan-Meier method to estimate the upgrade hazard rate (Figure 3-a). Model-free estimations show that in the first month after release and, to a less extent, the holiday season, existing consumers are more likely to upgrade. The upgrade hazard rate decreases over time and drops close to 0 one year from the date of release (Figure 3-a), after which a newer generation is released.

![Figure 3. Kaplan-Meier Estimation of Hazard Rate and Survival Function](image)

We also compare the survival probabilities of leapfroggers and switchers using three methods, Kaplan-Meier, Cox, and Weibull. As shown in Figure 3-b, switchers have a lower survival probability, or equivalently, a higher hazard rate under all three models. This implies that switchers, who have bought the immediate past generation, tend to upgrade to the new generation earlier. On average, it takes all existing consumers around 3.9 months to upgrade to a new generation. For switching consumers, it takes 2.46 months to upgrade, while for leapfrogging consumers, upgrades can take 4.92 months on average. This result might be counterintuitive to some, but is consistent with theoretical and empirical findings of the prior literature (Jiang and Jain 2012). In our empirical analysis, the difference between switching and leapfrogging consumers is modeled through the Switch dummy variable.

5. Empirical Evaluations

In this section, we use the game adoption and usage data described in the previous section to (i) examine whether consumers’ past adoption and usage experience can help explain consumers’ time to upgrade to a
new game generation, and (ii) evaluate the prediction performance of the Expo-Decay model in relation to benchmark models.

5.1. Model Estimation and Behavioral Explanations

We estimate the proportional hazard models with different specifications. Two dummy variables, corresponding to the first month after release and the holiday month, respectively, are included to capture the abnormalities. The estimation results are summarized in Table 4. From this table, we can see that the results produced by the six compared models are quite consistent—they all show that all intrinsic experience-based variables have significant associations with existing players’ time to upgrade decisions. In terms of model fitting, as measured by the Bayesian Information Criterial (BIC), the proposed Expo-Decay model fits the data the best among all methods. The superior model fitting performance, coupled with the fact that it is the most parsimonious and interpretable among comparable models, makes Expo-Decay the model of choice in our subsequent exploration.

We first take a look at consumers’ propensity to product upgrade. The scale parameter ($\gamma$) represents the initial upgrade hazard rate at the release date. Based on the definition of the Expo-Decay model in Eq. (5), when the new product generation is released ($t=0$), the instantaneous initial upgrade probability $h(t = 0) = \gamma = 0.1292$, or around 13%. The positive decay rate ($\alpha$) reflects a declining upgrade hazard trend, according to which the upgrade hazard decreases by 15.5% after waiting for an additional month.

We next examine how consumers’ past adoption and usage experience can help predict their time to product upgrades. Note that although the absolute values of estimated coefficients vary due to different model specifications, the signs of coefficients and significance levels are highly consistent. The results in Table 4 confirms that consumers’ previous adoptions are indeed significant predictors of their future upgrade decisions. Specifically, the positive coefficient for $\text{NumGens}$ suggests that, for consumers who

---

3 Specifications with no dummy variable or either one of the dummy variables have also been tested and the results are quantitatively consistent.
4 The declining rate is calculated as $\frac{h(t)-h(t-1)}{h(t-1)} = e^{-\alpha} - 1 = -0.155$. 

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have adopted one more generation in the game series, their hazard rates to upgrade at any given point in time increase by a factor of \( \exp(\beta_{\text{NumGems}}) = e^{0.4479} = 1.565 \) or by 56.5% on average. The negative influence of \( \text{WaitDays} (-0.0015) \) is also consistent with the literature—early adopters tend to upgrade earlier. Furthermore, the positive coefficient for \( \text{Switch} (0.3329) \) confirms that potential switching consumers, those who have adopted the immediate past generation, are about 40% \( (e^{0.3329} = 1.395) \) more likely to upgrade compared to potential leapfrogging consumers, i.e., those who did not purchase the immediate past generation. These results suggest that consumers who spend more on previous game generations also tend to upgrade earlier; intuitively, they represent the group of loyal customers and bigger spenders.

**Table 4. Estimation Results of Different Proportional Hazard Models**

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Cox Coefficient (Std.)</th>
<th>Weibull Coefficient (Std.)</th>
<th>Expo-Power Coefficient (Std.)</th>
<th>Restricted Box-Cox Coefficient (Std.)</th>
<th>Expo-Decay Coefficient (Std.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{NumGens} )</td>
<td>0.4352 (0.0142) ***</td>
<td>0.4565 (0.0141) ***</td>
<td>0.4483 (0.0143) ***</td>
<td>0.4483 (0.0146) ***</td>
<td>0.4479 (0.0143) ***</td>
</tr>
<tr>
<td>( \text{WaitDays} )</td>
<td>-0.0016 (0.0001) ***</td>
<td>-0.0015 (0.0001) ***</td>
<td>-0.0015 (0.0001) ***</td>
<td>-0.0015 (0.0001) ***</td>
<td>-0.0015 (0.0001) ***</td>
</tr>
<tr>
<td>( \text{Switch} )</td>
<td>0.3066 (0.0328) ***</td>
<td>0.3379 (0.0335) ***</td>
<td>0.3300 (0.0143) ***</td>
<td>0.3300 (0.0334) ***</td>
<td>0.3329 (0.0333) ***</td>
</tr>
<tr>
<td>( \text{NumSess} )</td>
<td>0.0016 (0.0001) ***</td>
<td>0.0026 (0.0001) ***</td>
<td>0.0025 (0.0001) ***</td>
<td>0.0025 (0.0001) ***</td>
<td>0.0025 (0.0001) ***</td>
</tr>
<tr>
<td>( \text{GiniIndex} )</td>
<td>-1.4165 (0.1738) ***</td>
<td>-0.9374 (0.1373) ***</td>
<td>-0.1965 (0.1895) ***</td>
<td>-0.1965 (0.2056) ***</td>
<td>-0.1965 (0.174) ***</td>
</tr>
<tr>
<td>( \text{EnhancePurchase} )</td>
<td>0.0012 (0.0003) ***</td>
<td>0.0016 (0.0003) ***</td>
<td>0.0017 (0.0003) ***</td>
<td>0.0017 (0.0003) ***</td>
<td>0.0017 (0.0003) ***</td>
</tr>
<tr>
<td>( \text{RecentActDay} )</td>
<td>-0.0028 (0.0001) ***</td>
<td>-0.0026 (0.0001) ***</td>
<td>-0.0027 (0.0001) ***</td>
<td>-0.0027 (0.0001) ***</td>
<td>-0.0027 (0.0001) ***</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.1119 (0.0116) ***</td>
<td>---</td>
<td>1.0744 (0.0317) ***</td>
<td>1.0744 (0.0317) ***</td>
<td>1.0744 (0.0317) ***</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.7649 (0.2841) **</td>
<td>0.1077 (0.0206) ***</td>
<td>0.1077 (0.0206) ***</td>
<td>0.1077 (0.0206) ***</td>
<td>0.1077 (0.0206) ***</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-0.1443 (0.0102) ***</td>
<td>---</td>
<td>-2.3222 (0.2015) ***</td>
<td>-2.3222 (0.2015) ***</td>
<td>-2.3222 (0.2015) ***</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>(0.2015) ***</td>
<td>---</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>(0.0506) ***</td>
<td>---</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>(0.1494)</td>
<td>---</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>(0.0032)</td>
<td>---</td>
</tr>
<tr>
<td><strong>BIC</strong></td>
<td>186796.9</td>
<td>56672.71</td>
<td>56320.72</td>
<td>56330.44</td>
<td>56315.81</td>
</tr>
</tbody>
</table>
Regarding consumers’ past usage experience, the positive coefficient for \(\text{NumSess} \ (0.0025)\) indicates that the heavy users, i.e., those who play more often, trend to upgrade earlier. Specifically, for consumers who have played one more game session of the previous generations, their average probability of upgrades increases by around 0.25%. The negative coefficient for \(\text{GiniIndex} \ (-1.3632)\) suggests that specialized players, those who spend most of their gaming time on a small subset of game modes (large \(\text{GiniIndex}\) indicating a low variety of usage), are less likely to upgrade to a new generation. This may appear to be counterintuitive at first glance. After much deliberation, our explanation is that specialized players, after exploring different game modes, might have already identified the game mode(s) they like the most, subsequently spend most of their time on a selected few game modes. As a result, they do not benefit as much from a product upgrade. In other words, a high Gini-Index may indicate the player is satisfied with features of the current game generation in use, which lowers her desire to upgrade to a new generation. This is analogous to the current trend in the smartphone industry — consumers’ satisfaction with old-generation smartphones postpones upgrades (Martin and FitzGerald, 2018). Another possible explanation is that, because they use a relatively small portion of the available game modes, specialized players might benefit less from an upgrade, hence are less likely to upgrade to a new generation.

Furthermore, the positive coefficient for \(\text{EnhancePurchase} \ (0.0017)\) reveals that consumers who have made more enhancement purchases in previous generations are more likely to upgrade. Although a relatively small proportion (around 18% according to our data) of players have ever made in-game purchases, they represent high-end consumers with a higher willingness to upgrade. Finally, the negative coefficient for \(\text{RecentActDay} \ (-0.0027)\) indicates that consumers who are active more recently (a smaller \(\text{RecentActDay}\)) have a higher probability to make upgrade purchases.

In sum, the results in Table 4 confirm that consumers’ prior adoptions and usage experience are significant predictors for their likelihood of upgrade and time to upgrade. It is important to note that the results of the statistical tests here indicate the strength of associations between the covariates and the hazard rate (or the likelihood of upgrade), or how strong a predictor each of the variables is. They do not suggest cause-effect relationships. In the dataset we use in this study, we believe the real potential causes
that affect consumers’ time-to-update decisions, such as their disposable income, affection for the game, amount of leisure time, willingness to take risk, or promptness in actions, are actually unobservable to firms. The observables are manifestations of these personal traits, not the real causes. In this particular application, there is limited downside even if the real causes are unobservable, because the observables are easy to extract from existing data resources, and are all that is needed to apply the models developed in this research to make predictions or help making marketing decisions.

5.2. Comparisons of Prediction Performance

Because a survival model predicts the distribution of time-to-event, the prediction performance is firstly evaluated at the aggregate level—by summing up all consumers’ predicted upgrade probabilities in each time period, we obtain the predicted aggregate upgrade sales, which is then compared against the actual aggregate sales in the same period. This way, we can compare the Expo-Decay model with benchmark methods in the prediction of aggregate sales. Since the Cox PH model does not specify a baseline hazard function and thus cannot generate an absolute upgrade probability, the comparison is conducted between the Expo-Decay model and parametric benchmark models, i.e. Weibull, Expo-Power, and restricted Box-Cox model.

We use 75% of unique players’ records to estimate the model, and the remaining 25% for out-of-sample test of prediction performance. Based on the values of the parameters estimated from the training sample, we calculate each individual record’s upgrade hazard rate based on Eq. (7), and thus the probability to upgrade, during each time interval in the testing set. By summing up the individuals’ probability to upgrade at each discrete time interval, we obtain the predicted monthly upgrade sales; the comparison with actual upgrade sales are summarized in Figure 4. The result shows that all four models produce predictions close to the actual quantities. We also observe that the predictions generated by the Expo-Decay model and the flexible Expo-Power and Restricted Box-Cox model are the closest.
To compare the prediction performances in a more systematic manner, we use four metrics, including Theil’s inequality coefficient, Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). With all four metrics, a small number indicates a better prediction. The Theil’s inequality coefficient (Theil 1961) is defined as:

$$U = \frac{\sqrt{\sum_{t=1}^{T} (y_t - \hat{y}_t)^2/T}}{\sqrt{\sum_{t=1}^{T} y_t^2/T} + \sqrt{\sum_{t=1}^{T} \hat{y}_t^2/T}}$$

(10)

in which $y_t$ and $\hat{y}_t$ are actual and predicted number of upgrades in month $t$. The coefficient $U$ ranges from 0 to 1, where a smaller value indicates a better prediction performance.

### Table 5. Comparison of Upgrade Sales Predictions

<table>
<thead>
<tr>
<th></th>
<th>Weibull</th>
<th>Expo-Power</th>
<th>Restricted Box-Cox</th>
<th>Expo-Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theil’s Coefficient</td>
<td>0.0269</td>
<td>0.0198</td>
<td><strong>0.0196</strong></td>
<td>0.0198</td>
</tr>
<tr>
<td>MSE</td>
<td>559.55</td>
<td>302.41</td>
<td><strong>297.68</strong></td>
<td>303.36</td>
</tr>
<tr>
<td>MAE</td>
<td>15.36</td>
<td>11.95</td>
<td>11.86</td>
<td><strong>11.27</strong></td>
</tr>
<tr>
<td>MAPE</td>
<td>74.74%</td>
<td>36.36%</td>
<td>35.86%</td>
<td><strong>35.24%</strong></td>
</tr>
</tbody>
</table>

From the comparison summarized in Table 5, we conclude that the Expo-Decay model performs as good as the Expo-Power model and the restricted Box-Cox model, and significantly better than the Weibull model. This result, coupled with the fact that the Expo-Power and the restricted Box-Cox models
are less parsimonious and are far more difficult to interpret, leads to the conclusion that the proposed Expo-Decay model is a superior predicting model.

Even though a distribution cannot be directly compared with a realized time, we can use the distribution of time produced by the Expo-Decay model to compute individual consumers’ probability of making an upgrade purchase during a prespecified time period, which is then comparable to the output of a classifier. To evaluate its performance at the individual level, we compare the prediction accuracy of the Expo-Decay model with that of a logistic regression using the area under a time-dependent receiver operating characteristic (ROC) curve (Bradley 1997). Figure 5 shows the area under the curve (AUC) for predicting an existing user making upgrade purchase one month after release, two months after release, and so on. Experiment results demonstrate that the proposed Expo-Decay model always dominates the benchmark logistic regression.

![Figure 5. AUC of Individual Upgrade Predictions (Expo-Decay vs. Logistic)](image)

With all empirical results considered, we conclude that the Expo-Decay model is an effective method in explaining and predicting existing consumers’ time-to-upgrade decisions.

### 6. Model Extensions

As a parsimonious model, the Expo-Decay model leaves open the possibility for model variations and extensions. In this section, we develop and evaluate three extension models to account for unobserved consumer heterogeneities, more detailed adoption history, and more frequent updates of variable values.
Table 6. Estimation Results for Extended Models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Expo-Decay Coefficient (Std.)</th>
<th>Frailty Expo-Decay Coefficient (Std.)</th>
<th>Double Expo-Decay Coefficient (Std.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NumGens</td>
<td>0.4479 (0.0143) ***</td>
<td>0.6538 (0.0260) ***</td>
<td>0.6729 (0.0150) ***</td>
</tr>
<tr>
<td>WaitDays</td>
<td>-0.0015 (0.0001) ***</td>
<td>-0.0015 (0.0001) ***</td>
<td>—</td>
</tr>
<tr>
<td>Switch</td>
<td>0.3330 (0.0334) ***</td>
<td>0.5006 (0.0460) ***</td>
<td>—</td>
</tr>
<tr>
<td>NumSess</td>
<td>0.0025 (0.0001) ***</td>
<td>0.0059 (0.0004) ***</td>
<td>0.0032 (0.0001) ***</td>
</tr>
<tr>
<td>GiniIndex</td>
<td>-1.3626 (0.1783) ***</td>
<td>-1.8418 (0.2395) ***</td>
<td>-1.7273 (0.2006) ***</td>
</tr>
<tr>
<td>EnhancePurchase</td>
<td>0.0017 (0.0003) ***</td>
<td>0.0107 (0.0019) ***</td>
<td>0.0020 (0.0003) ***</td>
</tr>
<tr>
<td>RecentActDay</td>
<td>-0.0027 (0.0001) ***</td>
<td>-0.0028 (0.0004) ***</td>
<td>-0.0028 (0.0001) ***</td>
</tr>
<tr>
<td>α</td>
<td>0.1687 (0.0039) ***</td>
<td>0.1476 (0.0042) ***</td>
<td>0.2069 (0.0084) ***</td>
</tr>
<tr>
<td>γ</td>
<td>0.1291 (0.0214) ***</td>
<td>0.1781 (0.0413) ***</td>
<td>0.1064 (0.0201) ***</td>
</tr>
<tr>
<td>σ²</td>
<td>—</td>
<td>0.9941 (0.0844) ***</td>
<td>—</td>
</tr>
<tr>
<td>α_{G−3}</td>
<td>—</td>
<td>—</td>
<td>0.0029 (0.0011) **</td>
</tr>
<tr>
<td>α_{G−2}</td>
<td>—</td>
<td>—</td>
<td>0.0040 (0.0015) **</td>
</tr>
<tr>
<td>α_{G−1}</td>
<td>—</td>
<td>—</td>
<td>0.0391 (0.0064) ***</td>
</tr>
<tr>
<td>φ_{G−3}</td>
<td>—</td>
<td>—</td>
<td>0.0271 (0.0088) **</td>
</tr>
<tr>
<td>φ_{G−2}</td>
<td>—</td>
<td>—</td>
<td>0.0479 (0.0114) ***</td>
</tr>
<tr>
<td>φ_{G−1}</td>
<td>—</td>
<td>—</td>
<td>0.1415 (0.0102) ***</td>
</tr>
<tr>
<td>BIC</td>
<td>56315.81</td>
<td>56059.33</td>
<td>56945.27</td>
</tr>
</tbody>
</table>

6.1. Frailty Expo-Decay Model

In reality, it is typical that only a proportion of players’ profile or shopping and usage history is observable, even though unobservable factors may have impacted existing players’ upgrade hazards as well. According to Jain and Vilcassim (1991), the empirical estimation might be biased in the absence of unobserved heterogeneity. To address this potential limitation, we introduce a random variable $\theta$ to the baseline hazard function to capture the unobserved consumer heterogeneity:

$$ h(t|X_i, \theta) = \theta \cdot h_0(t) \cdot e^{X_i \beta}, \quad (11) $$
We assume that $\theta$ follows a gamma distribution with an expected value of 1, $\theta \sim Gamma(\frac{1}{\sigma^2}, \frac{1}{\sigma^2})$, $E(\theta) = 1$, and $Var(\theta) = \sigma^2$. Correspondingly, the survival function takes the form of

$$S(t|X_i, \theta) = \exp\{-\int_0^t \theta * h_0(\tau) * e^{X_i \beta} d\tau\}. \quad (12)$$

This model is termed as Frailty Expo-Decay model. From the empirical estimation results summarized in Table 6, we can see that this model extension does improve model fitting. By introducing just one more parameter, the Frailty Expo-Decay model provides the best model fitting (in term of BIC).

However, the Frailty Expo-Decay model does not perform as well as the baseline Expo-Decay model in predicting aggregate upgrade sales (see Table 7) and individual upgrade timing as measured by AUC (see Figure 6). This comparison shows that, despite the existence of unobservable covariates, aggregate measures of players’ past adoption and usage experience can be effective predictors of their future upgrade decisions. It is possible that, in an attempt to capture the unobservable consumer heterogeneity, the Frailty Expo-Decay model leads to overfitting, thus hurting its prediction performance.

### Table 7. Aggregate Upgrade Sales Prediction

<table>
<thead>
<tr>
<th>PHMs</th>
<th>Expo-Decay</th>
<th>Frailty Expo-Decay</th>
<th>Double Expo-Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theil's Measure</td>
<td>0.0198</td>
<td>0.0210</td>
<td>0.0207</td>
</tr>
<tr>
<td>MSE</td>
<td>303.36</td>
<td>334.45</td>
<td>325.5</td>
</tr>
<tr>
<td>MAE</td>
<td>11.27</td>
<td>12</td>
<td>11.68</td>
</tr>
<tr>
<td>MAPE</td>
<td>35.24%</td>
<td>36.63%</td>
<td>37.30%</td>
</tr>
</tbody>
</table>

Figure 6. AUC of Individual Upgrade Predictions by Extension Models
6.2. Double Expo-Decay Model

In the baseline Expo-Decay model, an existing player’s previous adoptions are aggregated into cumulative variables (e.g. NumGens and WaitDays) to reflect the player’s experience with the product series. If a more refined view is taken, consumers’ adoption experience with more recent generations might be a stronger predictor of future upgrade decisions than their experience with older generations. For instance, if one consumer adopted three previous generations 7, 6, 5 years ago, respectively, and another consumer adopted three previous generations 4, 2, 1 years ago, respectively, although NumGens = 3 in both cases, it can be argued that the two consumers’ past adoption experience are not the same. Therefore, instead of using aggregated variables to represent a player’s adoption history, we propose an extension model that directly captures consumers’ discrete adoption behaviors while assuming that the predicting power of these adoption events decays over time. Mathematically, the extension model is characterized by the following hazard rate:

\[
h(t, X_i) = h_0(t) e^{X_i' \beta} + \sum_{g=1}^{G-1} 1_{i,g} * \alpha_g * e^{-\phi_g * (t - \tau_{i,g})},
\]

where \(1_{i,g}\) indicates whether consumer \(i\) has adopted generation \(g\), \(\alpha_g\) measures the influence of the adoption of generation \(g\) with a decaying factor \(\phi_g\), and \(\tau_{i,g}\) denotes the time at which player \(i\) activated generation \(g\). Given this additional decaying factor, we call this extension model the Double Expo-Decay model.

The empirical result in Table 6 for the Double Expo-Decay model confirms that adoptions of previous game generations show different level of association with future upgrade timing: the activations of more recent game generations have a stronger association with future upgrade timing, although their strength of association also decays faster. However, the Double Expo-Decay model does not seem to improve on model fitting in terms of BIC. This is because, although modeling previous adoption events individually minimizes errors in model fitting, the BIC measure penalizes the extension model because of the newly introduced parameters. Compared to the Double Expo-Decay model, the baseline Expo-Decay
model can adequately explain consumers’ upgrade behaviors without modeling each individual adoption event.

In addition to the aggregate sales, we also evaluate the models’ prediction performance for individual player’s upgrade decisions. As we can see from Table 7 and Figure 6, the Double Expo-Decay model is inferior to the baseline model in terms of prediction accuracy. We again believe that this is due to overfitting resulting from applying the more complicated model.

6.3. The Time-Variant Model Extension

For the baseline Expo-Decay model as well as the two extension models discussed so far, the values of the predictor variables are obtained using existing players’ adoption and usage data up to the release date of the focal new generation. In other words, even if a player continues to use a previous generation after the release date of the focal generation, the additional usage data is not used in model fitting and prediction. Apparently, if a firm would like to continuously analyze its market and make predictions, the methods we have used thus far would be limited. For instance, in our data, one player adopted G-13 and G-14 1.4 months and 4.9 months after their respective release dates. After G-15 is released, the estimated upgrade timing by the baseline model is around 6 months. However, we find that the player adopted G-12 and played G-12 for a few sessions. The player eventually made her upgrade purchase of G-15, but one year after its release date.

In settings where analysis and prediction are needed after the release date of a new generation, we propose a time-variant extension to the Expo-Decay model, in which the variables reflecting players’ experience are time-variant:

\[ h(t, X_i(t)) = h_0(t) \cdot \exp \left[ X_i(t) \beta \right], \]

and the corresponding survival function becomes

\[ S(t, X_i(t)) = e^{- \int_0^t h_0(u) \exp \left[ X_i(u) \beta \right] \, du}. \]

Analogous to the baseline Expo-Decay model, we propose a discrete version of the model for use with discrete-time data:
$S(t, X_i(t)) = e^{-\sum_{u=1}^{t} \left( \exp^{X_i(u)f} \cdot \int_{u-1}^{u} h_0(\tau) \, d\tau \right)}$.  

(16)

The experience-based variables, $X_i(t)$, are recomputed at the beginning of each discretized time interval and then used to compute the hazard rate for the next interval. While the variables are being updated over time, the coefficients $\beta$ are fixed.

We repeat our empirical testing and compare the performance of time-variant models with those of time-static models. The results are summarized in Table 8. As shown in the table, unlike the first two model extensions, the time-variant extension does lead to a better model fitting result, reflected by a lower BIC value.

### Table 8. Estimation Results for Time-Variant Models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficient (Std.)</th>
<th>Coefficient (Std.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-Static Model</td>
<td>Time-Variant Model</td>
<td></td>
</tr>
<tr>
<td>NumGens</td>
<td>0.4479 (0.0143) ***</td>
<td>0.4436 (0.0143) ***</td>
</tr>
<tr>
<td>WaitDays</td>
<td>-0.0015 (0.0001) ***</td>
<td>-0.0016 (0.0001) ***</td>
</tr>
<tr>
<td>Switch</td>
<td>0.3330 (0.0334) ***</td>
<td>0.3228 (0.0314) ***</td>
</tr>
<tr>
<td>NumSess</td>
<td>0.0025 (0.0001) ***</td>
<td>0.0013 (0.0001) ***</td>
</tr>
<tr>
<td>GiniIndex</td>
<td>-1.3626 (0.1783) ***</td>
<td>-1.6862 (0.1904) ***</td>
</tr>
<tr>
<td>EnhancePurchase</td>
<td>0.0017 (0.0003) ***</td>
<td>0.0022 (0.0003) ***</td>
</tr>
<tr>
<td>RecentActDay</td>
<td>-0.0027 (0.0001) ***</td>
<td>-0.0026 (0.0001) ***</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.1687 (0.0039) ***</td>
<td>0.1228 (0.0041) ***</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1291 (0.0214) ***</td>
<td>0.1900 (0.0333) ***</td>
</tr>
<tr>
<td>BIC</td>
<td>56315.81</td>
<td>56305.76</td>
</tr>
</tbody>
</table>

We also compare the prediction accuracy of the time-variant Expo-Decay model with that of the baseline Expo-Decay model, i.e., the time-static Expo-Decay model. Since the time-variant model requires variables to be updated overtime, it is not appropriate to be applied in forecasting upgrade sales at the release date. Thus, we evaluate the performance of time-variant model on predicting individual
player’s upgrade decisions using time-dependent ROCs. As shown in Figure 7, the time-variant Expo-Decay model outperforms the static Expo-Decay model in prediction accuracy.

![Graph: AUC of Individual Upgrade Predictions by Extended Models](image)

**Figure 7. AUC of Individual Upgrade Predictions by Extended Models**

6.4. Model Complexity vs. Data Quality

In the comparison between the Expo-Decay model and existing benchmark models, we find that the simpler Expo-Decay model performs as well as or better than more complex models. The comparison between the Expo-Decay model and the first two extension models leads to a similar observation—models with higher complexity or more parameters do not necessarily lead to better performance. On the other hand, our fourth test shows that the time-variant Expo-Decay model, which essentially keeps the original model formulations but uses more up-to-date data, performs clearly better than the baseline Expo-Decay model that does not update the values of predictor variables.

These comparisons lead to a conclusion we would like to highlight—for predictive analytics, data quality could be more important than model complexity in helping improve prediction performance. Therefore, instead of overspending on developing more complicated models, it might be more cost effective to invest in improving data quality.

7. Concluding Remarks

Continuous quality improvements and frequent releases of new generations in a product series is a common practice by businesses, which helps them counter competition, generate upgrade purchases, and
maintain market share. In the presence of successive product generations, it is important to understand consumers’ upgrade decisions when a new product generation becomes available. In this study, we are interested in an increasingly common scenario, where pre-release virtual adoptions account for a significant proportion of upgrade sales of a new product generation, and the upgrade hazard rate exhibits a declining pattern after the product release. We propose an Exponential-Decay Proportional Hazard Model (Expo-Decay model) to examine how existing consumers’ past experience (i.e. adoption and usage behavior) helps predict their time-to-upgrade decisions.

This study makes an important methodological contribution to the existing survival analysis literature. Specifically, the Expo-Decay model we propose can help explain the declining upgrade hazard rate of a new product generation when pre-release virtual adoptions account for a significant portion of upgrade sales. The Expo-Decay model is parsimonious, easy to interpret, and delivers excellent model fitting and prediction performance compared to existing parametric proportional hazard models, hence it has the potential of wide application in future academic research. Furthermore, we propose two time-static model extensions and one time-variant extension for use in different business settings. Our test results show that more complex model formulations do not lead to better prediction performance than the baseline Expo-Decay model, while the time-variant extension that updates the values of covariates over time outperforms the baseline Expo-Decay model with static data. These model extensions provide additional ready-to-use specifications for future research and practice applications.

This study also contributes to our understanding of the factors that help predict consumers’ time to product upgrade. Although the existing literature have identified some drivers that may influence consumers’ upgrade decisions, this study fills a void by linking consumers’ past adoption and usage experience to future upgrade purchases. Using a rich dataset for a video game series, we find that consumers’ prior adoptions and usage experience are significant predictors for their likelihood of upgrade and time to upgrade purchase. In particular, we find that (i) after a new product generation is released, potential switching consumers who are using the immediate past generation are more willing to upgrade;
(ii) consumers who use previous product series more often tend to upgrade earlier; and (iii) consumers
who specialized in a small subset of product features demonstrate a higher upgrade probability.

The proposed survival model and the empirical findings also have important managerial
implications. The Expo-Decay model can be used to predict future upgrade sales, which can help a firm
better manage the production, promotion, and distributions of a new product generation. The findings
regarding how consumers’ prior adoption and usage experience affects their time-to-upgrade can help
firms segment their market, design and deliver more specialized products for different types of
consumers, and develop personalized promotions to target consumers. Such personalized marketing
efforts can increase consumer satisfaction and improve the efficiency of operations, leading to better and
longer-term profitability.

This current study is not without limitations, which suggests several interesting future research
directions. First, we validate the proposed Expo-Decay model using video games dataset only. A future
study could test the model for other product categories and possibly develop a more specialized model
based on observed sales growth patterns. Second, we do not consider the impact of marketing mix
variables, such as price and promotion, on consumers’ time-to-upgrade decisions. It could be interesting
to extend the proposed Expo-Decay model to capture the impact of marketing mix variables. Third, one
could check the various information channels (e.g., social media) through which consumers can collect
information about a new product generation, and examine whether different information channels affect
consumers’ upgrade decisions differently.
References


