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Heterogeneous Firms

Jiahua CHE*

China Europe International Business School (CEIBS)

Wen ZHOU

University of Hong Kong

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* Corresponding author: Jiahua Che (jiahuache@ceibs.edu). Address: Department of Economics and Decision Sciences, China Europe International Business School (CEIBS), 699 Hongfeng Road, Pudong, Shanghai 201206, China.

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Abstract

We show that, independent of entry/exit a la Hopenhayn and market size expansion, trade with firm heterogeneity always crowds out less productive firms when countries are symmetric. When countries are asymmetric, however, trade can crowd in less productive firms and less productive firms almost always specialize in trade. We analyze how a country's standing in the world determines whether and how these phenomena will arise. Our paper helps reconcile empirical findings that are contradictory to the existing theoretical literature, and highlights the importance of country heterogeneity in understanding trade with firm heterogeneity.

Keywords: Firm heterogeneity, Asymmetric Trade, Reverse-Selection, Crowd-In, Country Standing

JEL codes: F10, F12, F14

*Jiahua Che: jiahua.che@gmail.com; Department of Economics and Decision Sciences, China Europe International Business School. Wen Zhou: wzhou@hku.hk; Faculty of Business and Economics, the University of Hong Kong. We thank Cheng Chen, Davin Chor, Giovanni Facchini, Wentai Hsu, Edwin Lai, Larry Qiu, Peter Neary, Chang Sun, Heiwai Tang, Hongsong Zhang, participants at GEP China Nottingham conference, 2018 Econometric Society Asian Meeting, 2018 EEA and ES European Meeting for helpful comments, Dingwei Gu for excellent research assistance, and China Europe International Business School for financial support. All errors are ours.

1 Introduction

This paper offers a simple framework to revisit the central finding of the literature on trade with heterogeneous firms, and presents a wealth of new insights that can help enrich our understanding of the literature and some of the conflicting empirical findings.

The past decade has witnessed an explosion of studies on trade with heterogeneous firms, thanks to pioneering works by Eaton and Kortum (2002), Melitz (2003), Bernard, Eaton, Jensen, and Kortum (2003), Bernard, Redding, and Schott (2007), Melitz and Ottaviano (2008), Chaney (2008), Arkolakis, Costinot, and Rodriguez-Clare (2012) among many others. Amidst its many remarkable insights, the hallmark contribution of this growing literature is its identification of new gains from trade: by crowding out less productive firms and allowing more productive firms to expand, trade reallocates resources from low productivity firms to high productivity firms, thus improving overall productivity in an economy (see Helpman (2006), Redding (2011), Melitz and Trefler (2012), and Melitz and Redding (2015) for reviews).

The literature had emerged in response to the empirical phenomenon that exporting firms tend to be more productive than non-exporting firms (see Wagner (2007) for a survey of the empirical findings), mostly because more productive firms self-select into exporting (see Bernard and Jensen (1999), Aw, Chung, and Roberts (2000), and Clerides, Lach, and Tybout (1998) for example). However, this seemingly robust pattern has recently been found to be at odds with what is happening in the largest exporting country in the world, and arguably the biggest beneficiary of globalization, China. Many have shown that Chinese exporters, especially those who are specialized in exports, are in fact less productive. Lu (2010), for example, noted that “China’s exporters are typically less productive and sell less in the domestic market than non-exporters.” Lu, Lu, and Tao (2010) also reported that “for foreign affiliates in China, exporters are found to be less productive.” Moreover, they found that “among foreign affiliates, those selling all their output in China have the highest productivity, followed by those having sales in China and also exporting some of their output, and finally those exporting all their output.” The same paradoxical pattern in China has been found by Dai, Maitra, and Yu (2016), Ma, Tang, and Zhang (2014), Manova and Yu (2016), and Chen and Sun (2019).

Can this reverse-selection (exporters being less productive) be more robust than a phenomenon unique to China? If so, can we reconcile such reverse-selection with the theoretical literature of trade with firm heterogeneity? In particular, if it is the less productive firms expanding to foreign markets, how should we understand the new source of gains from trade in which trade is supposed to crowd out less productive firms and reward the foreign markets to the more productive firms alone?

To address these questions, we set up a simple model of monopolistic competition with heterogeneous firms. Featuring a general class of additive separable preferences with finite marginal utility, our model is able to reproduce the key insights of the literature, i.e., trade among symmetric countries crowds out less productive firms and allows the more productive firms to expand to foreign markets, without resorting to any fixed costs.

Our simple model highlights two departures from the existing literature. First, it demonstrates that these key insights hold so long as countries are symmetric, and are independent of free entry *à la* Hopenhayn (1992), a key ingredient to the existing literature. We show that trade can make each and every firm less profitable, so that trade leads to either no entry or exodus of firms. Even when trade does raise firm profitability, entry only intensifies, but does not cause, the crowding-out. In other words, more productive incumbent firms crowd out less productive firms in symmetric trade neither as a result of, nor necessarily resulting in, any new entrants.

Second, we show that trade is not tantamount to an expansion in market size. While trade among symmetric countries always crowds out less productive firms, expanding a single country's market size can crowd in less productive firms. We point out that there exists a fundamental difference between trade and market size expansion: trade brings new varieties from abroad, whereas size expansion does not.

Accordingly, our simple model demonstrates that the new gains from trade have nothing to do with size expansion or scale economy (as in Krugman 1979). Instead, the gains originate from the fact that trade brings new varieties that are produced in foreign countries with their more productive firms. Without trade, any new variety must be produced domestically by marginal and hence less productive firms. With trade, each country will expand the production of their more productive firms and use these additional outputs to exchange for new varieties from abroad.

When countries are asymmetric, a much richer trade pattern arises. Two features are particularly noteworthy. First, in larger countries or countries with lower overall productivity, less productive firms always specialize in exporting. This is because trade balance entails that, in equilibrium, a larger country or a less productive country must have a lower per capita earnings than their trading partners. In such poorer countries, therefore, foreign demand is stronger than domestic demand, so their least productive firms can survive only in foreign markets but not at home.

Second, when countries are asymmetric, trade can crowd in, rather than crowd out, less productive firms. As alluded above, trade amounts to acquiring a new variety by having a domestic firm to produce for the foreign market. When countries are asymmetric, it is entirely possible for the foreign value to exceed the domestic production cost of even the marginal variety,

even though the domestic value is always smaller than the domestic cost. In such a case, further trading continues to generate positive social gains, and there must exist a term of trade to make this efficiency-enhancing trade mutually beneficial and consequently happen in equilibrium. Such a scenario never arises in symmetric trade, where foreign value always equals domestic value by symmetry.

In sum, with firm heterogeneity, trade begins because of gains from expanding more productive firms to exchange for new varieties from abroad. As gains do not stop there, neither does trade. When countries are asymmetric, gains from trade can be further attained by bringing into action even the less productive firms in some countries that would otherwise not operate under autarky. It now becomes clear that such crowding-in, if it takes place, must occur in (endogenously) poorer countries in the presence of (endogenously) richer countries. In other words, crowding-in in poorer countries complement crowding-out in richer countries, where the gains always take the form of expanding more productive firms at the expense of less productive firms.

Our analysis has an important implication. That is, the apparent robust empirical pattern that has inspired the literature of trade with heterogeneous firms may be more relevant for some countries than for others. In particular, when it comes to trade with firm heterogeneity, gains from trade can take different forms to materialize in different countries. How these countries differ depends on the structure of the global economy, which in turn is endogenously determined according to exogenous differences in country size or overall productivity. Therefore, a deeper understanding of trade with firm heterogeneity requires treating the world as an asymmetric system rather than a symmetric one.

Our paper is related to a number of efforts aimed at expanding the literature on trade with heterogeneous firms. Most of these efforts have focused on expanding the preferences for the tradable goods beyond (but inclusive of) constant elasticity of substitution preferences. For example, Zhelobodko et al. (2012) consider additive separable preferences with variable elasticity of substitution. They show that trade will crowd out less productive firms if consumers' "relative love for variety" is increasing in their consumption, and vice versa.¹ Mrázová and Neary (2017) take a different approach. By characterizing demand in terms of its elasticity and convexity, they show that trade will crowd out less productive firms if and only if the demand is "subconvex".²

Our paper differs from these analyses. First, our model is independent of free entry *a la* Hopenhayn (1992), which is instrumental to Zhelobodko et al. (2012) and Mrázová and Neary

¹Using the same form of preferences, Dhingra and Morrow (2019) identify a sufficient condition for trade to be welfare enhancing.

²According to Mirazova and Neary (2017), a function $p(x)$ is superconvex at a point (p_0, x_0) if and only if $\log p$ is convex in $\log x$ at (p_0, x_0) , and a function is subconvex if it is not superconvex.

(2017). Second, these two papers focus on symmetric countries. In contrast, we consider both symmetric and asymmetric countries. Third, in Zhelobodko et al. (2012) and Mrázová and Neary (2017), less productive firms may be crowded in because certain preference structure allows trade to affect their profitability more favorably vis-a-vis the more productive firms. In our model, trade crowds in less productive firms only when countries are asymmetric, and it does not depend on the preference structure.

Trade with heterogeneous firms among asymmetric countries was also analyzed by Chaney (2008) and Melitz and Ottaviano (2008). Both studies assume the presence of a tradable numeraire good that is produced in different countries with identical technology and at positive quantities in a trade equilibrium. In contrast, our model does not adopt these assumptions, and incomes differ endogenously among trading economies. This allows us to contribute new insights to the existing literature.

There have been a few attempts to address the reverse-selection phenomenon. Lu, Lu, and Tao (2010) maintain that specialization in export helps save fixed cost necessary for penetrating the Chinese market, i.e., exporting is a way for less productive firms to remain competitive. Using a similar logic, Lu (2012), Dai, Maitra, and Yu (2016), and Chen and Sun (2019) assume a higher cost to access the Chinese domestic market, which induces less productive firms to focus on the export market.³ These arguments essentially stick to Melitz (2003) with some modified assumptions. Instead of entangling with the question of which country requires a higher access cost, our model assumes away trade cost entirely. In our model, reverse selection takes place because of (endogenous) income differences, so that a larger and less productive country like China will in equilibrium find its less productive firms specialized in exports.

The rest of the paper is organized as follows. In the next section, we present our simple model of trade among symmetric countries, assuming first fixed labor supply and then endogenous labor supply to the tradable sector. Section 3 discusses the role of free entry, differences between trade and market expansion, and the driving force behind the gains from trade with heterogeneous firms. Section 4 analyzes trade among asymmetric countries, where countries may differ in population size or overall productivity. Section 5 concludes.

2 The Model

A representative economy can produce a continuum of varieties in a tradable sector. In particular, variety $j \in [0, \infty)$ is produced by a monopolistic firm at marginal cost $c(q, j)$ that is

³See also Melitz and Redding (2015) in their characterization of Lu (2012).

weakly increasing in q and strictly increasing in j with $\lim_{j \rightarrow \infty} c(0, j) = \infty$ (see Zhelobodko et al. (2012) for a similar assumption on the marginal cost). In other words, we sort varieties *produced* (and hence their corresponding firms) in a country according to the production cost, so the j th variety *produced* refers to the j th least costly variety available in that country.

The economy has a unit measure of homogeneous consumers, who also collectively own all the firms and who supply the only factor of production, labor. Each consumer supplies his labor at a constant marginal disutility that is normalized to one. An individual consumer's preference is represented by

$$U(x(i)_{i \in (0, \infty)}) - l \equiv f\left(\int_0^\infty u(x(i)) di\right) - l,$$

where the first term is utility derived from the tradable sector, with $x(i)$ being the consumption quantity of variety i , $u(\cdot)$ the utility from a given variety, $f(\cdot)$ the utility from the consumption basket of all tradable goods, $\{x(i)\}_{i \in [0, \infty)}$, and l is the amount of labor used in producing tradable goods, with the marginal cost equal to one. The cost of production, $-l$, can be interpreted either as the (lost) consumption of leisure time, or as the lost utility from producing and consuming a non-tradable good. We assume that $u' > 0$, $u'' < 0$, and $f' > 0$. In other words, consumers love varieties.

In addition, we impose a number of properties for our preferences, which we shall introduce one by one as our analysis requires. Our first property assumes u to have finite marginal utility.

Property (1*) $u' < \infty$.

This property sets our preferences apart from the constant elasticity of substitution (CES) preferences, for example. In the case of CES preferences, $u(\cdot)$ satisfies the Inada condition: $u'(0) = \infty$, making it impossible to analyze how trade impacts the absence of varieties, which naturally involves corner solutions, without invoking fixed cost. In contrast, our preferences enables us to do away with fixed cost in our analysis.

Since varieties enter a consumer's utility symmetrically, consumption quantity $x(i)$ varies across varieties only due to their price differentials. Provided that each variety's price increases monotonically in cost (which will be shown to be indeed the case), varieties sorted in an ascending order in production cost corresponds to varieties sorted in a descending order in consumption quantity. In autarky, therefore, the i th variety in a consumer's consumption basket corresponds to the i th least costly variety in the economy. In an open economy, a consumer sources his consumption in the global market, and such correspondence must be amended, as will be shown in a moment.

We assume that trade is costless, and that the sets of varieties produced by different countries

do not overlap.⁴ Following the literature, we assume monopolistic competition with uniform pricing within each country.

We will consider two types of labor supply: a fixed supply of labor to the tradable sector, and an endogenous allocation of labor between the tradable and non-tradable sectors. These two settings are the two major approaches adopted in the literature. Our analysis of both settings reveals a unifying fundamental force in trade with firm heterogeneity as well as how they may differ.

2.1 Fixed Labor

We begin with the fixed supply of labor. Consider trade among m symmetric economies. Let $q(j)$ denote the amount of the j th variety *produced in a country and sold in its domestic market*. In a symmetric equilibrium,⁵ variety j 's total production and global sales is $m q(j)$. Since a consumer sources his consumption from the global market, the i th variety *consumed* becomes the $j = \frac{i}{m}$ th variety *produced* in each country:

$$x(i) = q\left(\frac{i}{m}\right).$$

Consequently,

$$U = f\left(\int_0^\infty u(x(i))di\right) = f\left(\int_0^\infty u\left(q\left(\frac{i}{m}\right)\right)di\right) = f\left(m \int_0^\infty u(q(j))dj\right),$$

where the last equality obtains after re-indexing $i = mj$. Consumer optimization is therefore

$$\max_{q(j)_{j \in (0, \infty)}} f\left(m \int_0^\infty u(q(j))dj\right) \text{ s.t. } \int_0^\infty mp(j)q(j)dj \leq l + \pi,$$

where $p(j)$ is the price of variety j , l is each consumer's endowment of labor,⁶ and π is the profit of all firms in a country, which is eventually shared equally among the country's consumers.

Let $\tilde{\lambda}$ be the usual Lagrangian multiplier. Consumer maximization becomes

$$\max_{q(j)_{j=0.. \infty}} f\left(m \int_0^\infty u(q(j))dj\right) + \tilde{\lambda} \left[l + \pi - \int_0^\infty mp(j)q(j)dj \right].$$

For any variety j such that $q(j) \geq 0$, consumer optimization yields: $u'(q(j)) = \frac{\tilde{\lambda}}{f'(m \int_0^\infty u(q(j))dj)} p(j)$.

⁴Such an assumption can be endogenized if consumers have distinctive preferences for varieties from different sources (Armington 1969). In monopolistic competition, then, each firm will produce a unique variety in order to avoid competition. Such justification was implicitly adopted in the literature as well as our model. As will be argued later, what is important is the fact that firms in different countries produce different varieties; exactly how and why they end up doing so is not essential to the discussion.

⁵For ease of exposition, we will present our analysis of a symmetric equilibrium here, while relegating the proof of non-existence of any asymmetric equilibrium to footnote 18.

⁶Since countries are symmetric here, wage rate can be normalized without any loss of generality.

Define $\lambda \equiv \frac{\bar{\lambda}}{f'(m \int_0^\infty u(q(j))dj)}$ as the shadow price of labor. Then the demand for variety j becomes $u'(q(j)) = \lambda p(j)$.

Firm j (the producer of variety j) chooses $p(j)$ to maximize

$$\pi(j) \equiv p(j)m q(j) - \int_0^{mq(j)} c(q, j)dq;$$

and the optimization yields $u'(q) + qu''(q) = \lambda c(mq(j), j)$. Let

$$r(q) \equiv u'(q) + qu''(q)$$

denote the variety-level marginal revenue.

Denote by κ the production threshold, i.e., the index of the last variety produced in each country. Then the consumption threshold (i.e., the index of the last variety consumed) is $m\kappa$. By definition, $q(\kappa) = x(m\kappa) \equiv 0$. At κ , then, we have $r(0) = \lambda c(0, \kappa)$.

In sum, the trade equilibrium under fixed labor is characterized by three unknowns, λ , κ , and $q(j)$ for all $j \in [0, \kappa]$, which are solved uniquely from the following three equations:

$$\frac{r(q(j))}{\lambda} = c(mq(j), j), \text{ for } j \in [0, \kappa] \quad (1)$$

$$\frac{u'(0)}{\lambda} = c(0, \kappa), \quad (2)$$

$$\int_0^\kappa \int_0^{mq(j)} c(q, j)dqdj = l. \quad (3)$$

Equation (1) equates the marginal revenue (generated from any of the symmetric countries), factored by the shadow price of labor, with the marginal cost.⁷ Equation (2) applies equation (1) to the threshold firm, which produces zero output. Equation (3) is the labor market clearing condition. It is derived from the binding budget constraint:

$$l = \int_0^\kappa mp(j)q(j)dj - \pi = \int_0^\kappa mp(j)q(j)dj - \int_0^\kappa \left[mp(j)q(j) - \int_0^{mq(j)} c(q, j)dq \right] dj.$$

The solution of the equilibrium can be easily decomposed into three steps. For any given shadow price of labor, the intensive margin (i.e., $q(j)$) is determined by equation (1), and the extensive margin (i.e., κ) by (2). These two will generate a total labor demand as a function of labor's shadow price, the equilibrium value of which will then be solved from (3).

Notice that an autarky equilibrium can be obtained from the trade equilibrium by setting $m = 1$.

⁷The derivation follows the convention that an infinitesimal firm in monopolistic competition does not incorporate the impact of its output on $Q \equiv m \int_0^\infty u(q(j))dj$ and $\bar{\lambda}$ (see Zhelobodko et al. (2012) and Mrázová and Neary (2017)).

In the rest of the paper when a trade outcome is explicitly compared with an autarky outcome, we use superscript m to indicate trade, and superscript c to indicate autarky. When discussing trade equilibrium for any general $m \geq 1$, of which autarky is a special case ($m = 1$), we do not attach superscripts.

To ensure a concave profit function, we introduce an additional property for our preferences to ensure decreasing marginal revenue whenever it is positive:⁸

Property (2*) $2u''(q) + qu'''(q) < 0, \forall q$ such that $u'(q) + qu''(q) > 0$.

We can then reproduce the key insights of Melitz (2003):

Proposition 1 *Suppose that labor supply to the tradable sector is fixed, and consumers' preferences satisfy properties (1*) and (2*). Then there exists a unique equilibrium in autarky and trade respectively. Compared to autarky, in the trade equilibrium,*

- a) *the least productive firms cease operation ($\kappa^m < \kappa^c$); and*
- b) *consumption of each domestic variety drops ($q^m(j) < q^c(j)$).*

Proof. To prove the existence and uniqueness of a symmetric equilibrium,⁹ note that $q(j)$ is strictly decreasing in λ and so is κ by the concavity of $u(\cdot)$. Hence, the left-hand side of (3) is a strictly decreasing function of λ . The rest is straightforward.

For comparative statics between trade and autarky, we show by contradiction that $\lambda^m > \lambda^c$. Suppose not: $\lambda^m \leq \lambda^c$. Then $\kappa^m \geq \kappa^c$, implying that, in order for the labor market clearing condition to hold under both autarky and trade, there must exist $j < \kappa^c$ such that $mq^m(j) \leq q^c(j)$. This in turn implies by (1) that

$$r(q^m(j)) = \lambda^m c(mq^m(j), j) \leq \lambda^c c(q^c(j), j) = r(q^c(j)).$$

Given Property (2*), this further implies that $q^m(j) \geq q^c(j)$. Contradiction.

Q.E.D.

⁸Properties (1*) and (2*) are met by CARA and quadratic preferences, for example. Behrens and Murata (2007) show that, in monopolistic competition models, CARA preferences can generate what they refer to as “pro-competitive” and “competitive limit” effects on profit-maximizing prices.

⁹ We present a sketch of proof here for why asymmetric equilibrium does not exist. Details can be found in Section 4 including the meaning of notations. Suppose there is an asymmetric equilibrium. In particular, country x has $\kappa_x^x > \kappa_x^y$. Then $\frac{w^x}{\lambda^x} > \frac{w^y}{\lambda^y}$. As the two countries face the same set of supplies in the global market, country x must spend more than country y . Trade balance requires x to earn more revenue than country y . Since the two countries face the same set of demand in the world, this is possible only if $w^x < w^y$ so that country x produces more than country y . Since the cost function is the same, labor market cannot be cleared in both countries.

2.2 Endogenous Labor

We now turn to the case where labor is endogenously supplied to the tradable sector. This setting is similar to Melitz and Ottaviano (2008) and Chaney (2008) with labor being allocated between tradable varieties and a numeraire good. There are two differences, however. First, Melitz and Ottaviano (2008) and Chaney (2008) assume the presence of a numeraire good that is tradable, and that the good is produced/consumed at positive quantities in equilibrium by every country using the same constant returns to scale technology. Accordingly, in their analyses, any trade imbalance in the non-numeraire goods is absorbed by a corresponding imbalanced trade in the numeraire good. In contrast, we assume l to be non-tradable. While such a modelling difference is inconsequential when countries are symmetric, as trade is always balanced under symmetry, balanced trade in the tradable sector will play a crucial role in our analysis of asymmetric trade in Section 4.

Second, our consumer preference for the tradable goods combines the additive separability of CES as in Melitz (2003) and Chaney (2008) together with the feature of finite marginal utility in Melitz and Ottaviano (2008)'s quadratic preferences. As in the case of fixed labor supply, it helps simplify our analysis.

Different from the fixed labor supply case where $f(\cdot)$, aside from being monotonically increasing, plays no role in our analysis, the curvature of $f(\cdot)$ matters in the endogenous labor supply case. We therefore in addition to Properties (1*) and (2*) add two more properties as follows:

Property (3*) $f'' < 0$;

Property (4*) $f' \rightarrow 0$ if $\int_0^\infty u(x(i))di \rightarrow \infty$ and $f' \rightarrow \infty$ if $\int_0^\infty u(x(i))di \rightarrow 0$.

Once again, we focus on a symmetric equilibrium while relegating the proof of non-existence of any asymmetric equilibrium to footnote 10. In a symmetric equilibrium, consumer optimization problem is

$$\max_{q(j) \geq 0} f\left(m \int_0^\infty u(q(j))dj\right) - l$$

subject to the budget constraint $\int_0^\infty mp(j)q(j)dj \leq l + \pi$. In equilibrium, the budget constraint is binding, so $l = \int_0^\infty mp(j)q(j)dj - \pi$. Plug this into the consumer's objective function, and the first-order condition leads to the demand function for any variety with non-negative output: $f'(Q)u'(q(j)) = p(j)$, where

$$Q \equiv m \int_0^\infty u(q(j))dj$$

is the consumption composite of tradable goods. Firm optimization is

$$\max_{q(j) \geq 0} \pi(j) \equiv f'(Q)u'(q(j))mq(j) - \int_0^{mq(j)} c(q, j)dq,$$

which leads to $f'(Q)r(q(j)) = c(mq(j), j)$.

In sum, the trade equilibrium under endogenous labor is characterized by three variables, Q , κ , and $q(j)$ for $j \in [0, \kappa]$, which are solved uniquely from the following three equations:

$$f'(Q)r(q(j)) = c(mq(j), j) \text{ for } j \in [0, \kappa], \quad (4)$$

$$f'(Q)u'(0) = c(0, \kappa), \quad (5)$$

$$\int_0^\kappa mu(q(j))dj = Q. \quad (6)$$

As in the case of fixed labor, here the equilibrium is solved in three steps: the intensive margin, the extensive margin, and the consumption composite.

We can now reproduce the key insight of Melitz and Ottaviano (2008).

Proposition 2 *Suppose that labor is endogenously supplied to the tradable sector, and consumers' preferences satisfy properties (1*) through (4*). Then there exists a unique equilibrium in autarky and trade respectively. Compared to autarky, in the trade equilibrium,*

- a) *the least productive firms cease operation ($\kappa^m < \kappa^c$); and*
- b) *consumption of each domestic variety drops ($q^m(j) < q^c(j)$).*

Proof. For the existence and uniqueness of a symmetric equilibrium,¹⁰ note that $q(j)$ is strictly decreasing in Q and so is κ by the concavity of $u(\cdot)$ and $f(\cdot)$ (Property (3*)). Hence, the left-hand side of (6) is a strictly decreasing function of Q . This, together with Property (4*), establishes both the existence and uniqueness.

For comparative statics between trade and autarky, we show by contradiction that $Q^m > Q^c$. Suppose not: $Q^m \leq Q^c$. Then $f'(Q^m) \geq f'(Q^c)$ per Property (3*), which in turn implies that $\kappa^m \geq \kappa^c$ according to (5). Since

$$\int_0^{\kappa^c} u(q^c(j))dj = Q^c \geq Q^m = \int_0^{\kappa^m} mu(q^m(j))dj > \int_0^{\kappa^m} u(mq^m(j))dj,$$

¹⁰ We present a sketch of proof for why there does not exist any asymmetric equilibrium. Suppose not. Let country x have $\kappa_x^x > \kappa_x^y$. Then $w^x f'(Q^x) > w^y f'(Q^y)$. As the two countries face the same set of supplies, country x must spend more than country y . Trade balance requires x to earn more revenue than country y . Since the two countries face the same set of demand, this is possible only if $w^x < w^y$, implying that $f'(Q^x) > f'(Q^y)$, which contradicts the fact that country x consume more because of larger $f'(Q^x)$.

there must exist some $j < \kappa^c$ such that $mq^m(j) < q^c(j)$. This in turn implies by (4) that

$$f'(Q^m)r(q^m(j)) = c(mq^m(j), j) \leq c(q^c(j), j) = f'(Q^c)r(q^c(j)).$$

Given Property (2*), this further implies that $q^m(j) \geq q^c(j)$. Contradiction.

Q.E.D.

3 Discussions

We have reproduced the key insight obtained in Melitz (2003) and Melitz and Ottaviano (2008), that trade (among symmetric countries) crowds out less productive firms, in a unified simple framework.

The intuition behind such result is straightforward in our model. Consider first the fixed labor case. Because countries are symmetric, their shadow price of labor, λ , which determines the profitability of any firm, must be the same across these countries. This shadow price is determined by the labor supply and demand. Trade increases the labor demand (the left hand side of equation (3) increases in m) for any given λ . Hence, the shadow price of labor must increase as a result of trade, making it unprofitable for the marginal firms to continue to operate (see equation (2)).

Next, consider endogenous labor. Because countries are symmetric, their consumption composite in the tradable sector, Q , must be the same across these countries. This consumption composite determines the marginal value of the tradable sector, which in turn determines the marginal revenue obtained by the threshold firm. Trade enlarges the consumption composite; as a result, the marginal value of the tradable sector diminishes given that $f(\cdot)$ is concave, thus forcing the threshold firm to cease operation.

Figure 1 highlights the intuition discussed above.

3.1 The Role of Our Assumptions

The simplicity of our analysis is attributed to our additively separable preferences together with the assumed properties. Like the standard CES utility function, the utility function $f(\int_0^\infty u(x(i))di)$ allows us to decompose a variety's marginal value into two complementary components: a variety-invariant part that depends only on the consumption composite of the tradable goods, $f'(Q)$, and a variety-specific part, $u'(q)$, that is independent of the consumption of other varieties.¹¹ As a result, a monopolistic firm's marginal revenue is similarly decomposed into two complementary

¹¹The additively separable preference adopted by, for instance, Zhelobodko et al. (2012), Mrázová and Neary (2017), and Dhingra and Morrow (2019).

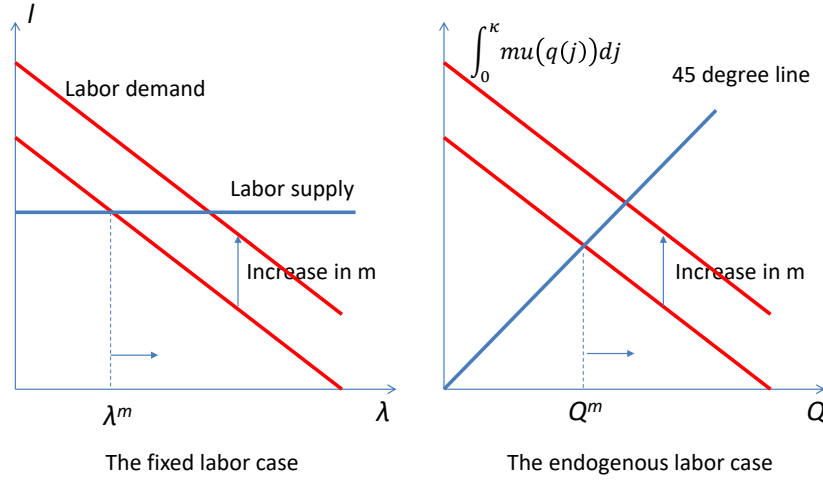


Figure 1: Trade raises the shadow price of labor (λ) and the consumption composite (Q)

components: a variety-invariant coefficient, and a variety-specific marginal revenue, $r(q) = u'(q) + qu''(q)$.¹² The coefficient is $f'(Q)$ in the case of endogenous labor, and $\frac{1}{\lambda}$ in the case of fixed labor. As the endogenous coefficient plays a crucial role in our model, we will refer to it as an *aggregator*.¹³ Trade raises the shadow price of labor (λ) and enlarges each consumer's consumption composite (Q), both of which will depress the aggregator and hence the demand for each individual variety that survives.

Unlike the standard CES utility function, our utility function is assumed to satisfy Property (1*). This enables us to determine the extensive margin (i.e., the threshold firm) by the zero output condition (a first-order condition). In contrast, when CES preferences are adopted as in, for example, Melitz (2003), some fixed cost of production becomes necessary to pin down the threshold firm by a zero profit condition. Together with the aforementioned decomposition of the marginal revenue, Property (1*) also allows us to express the intensive margin (i.e., the output of each operating firm) as a function of the aggregator. These two margins determine the labor demand and consumption composite, and finally the equilibrium value of the aggregator is solved from the labor market clearing condition (in the case of fixed labor) or the identity condition (in the case of endogenous labor).

Property (3*) says our consumers exhibit diminishing love of variety. When labor is endogenously supplied, this property (along with Property (2*)) offers a sufficient condition for the marginal firm to be crowded out in trade. As Q increases, the aggregator decreases and, as a result, the marginal value of the tradable sector as a whole declines. Since the marginal value

¹²See Mrázová and Neary (2017) for a more elaborated and general discussion on such decomposition.

¹³Such an aggregator corresponds to marginal utility of income in Mrázová and Neary (2017), and consumer's budget multiplier in Dhingra and Morrow (2019).

of the non-tradable sector is constant given our quasi-linear structure, the least productive firm must be crowded out from the tradable sector as a result. In this case, resources are reallocated both across firms within the tradable sector and between the tradable and non-tradable sectors, to accommodate the expansion of intra-margin firms into the foreign markets.¹⁴

Property (3*) is also necessary for trade to expand the tradable sector consumption ($Q^m > Q^c$) and crowd out the marginal firm at the same time. If $f''(.) > 0$, which we may refer to as increasing love for variety (as in the case of a standard CES preferences), then one can tell immediately from (5) that, should $Q^m > Q^c$, trade will attract less productive firms into operation instead of driving them out. This is because, as trade expands the tradable sector, increasing love for variety brings about a rising marginal value of labor in the tradable sector, making it more efficient to reallocate resources from the non-tradable sector to the tradable sector.

When labor is fixed in supply, labor never flows between the tradable and non-tradable sectors, and hence the marginal value of the tradable sector relative to the non-tradable sector becomes irrelevant for resource allocation. As a result, trade crowds out marginal firms regardless of the shape of $f(.)$ (provided that $f'(.) > 0$) and hence independent of Property (3*). This implies that whether or not consumers have increasing or diminishing love of variety is irrelevant when labor is in fixed supply. Rather, it is the labor demand as a function of its shadow price, λ , that determines how trade reallocates labor within the tradable sector. As trade opens up a better opportunity for an intra-margin firm to tap into the foreign market than for the marginal firms, the fixed amount of labor resource is competed away from the marginal firms by intra-margin firms (via an increase in the shadow price of labor, λ) to accommodate the latter's global expansion.

Throughout the rest of the paper, unless otherwise specified, our consumer preference satisfies Properties (1*) and (2*) for the fixed labor case, and Properties (1*) through (4*) for the endogenous labor case.

3.2 A Visit to the Hopenhayn Mechanism

Although we obtain the same insights as those established in the literature for symmetric trade, resource reallocation takes place in our model through a different mechanism. The literature relies crucially on entry/exit *a la* Hopenhayn (1992), which is irrelevant to our model. In this subsection, we extend our model to incorporate the possibility of Hopenhayn entry/exit to better understand the difference between the two mechanisms.

Imagine that a potential entrant can incur a fixed entry cost, f_e , to enter the tradable sector with a

¹⁴Property (3*) ensures that such an expansion also induces the non-tradable sector to compete resources away from the marginal firm of the tradable sector, leading to the marginal firm being crowded out in trade.

random productivity draw. For expositional simplicity, let's assume that the cost is psychological, in a form of stress for example, hence having no effect on labor resource allocation, and our previous analysis remains intact despite the presence of such cost. Our key results remain intact qualitatively even if the entry cost requires resources (see footnote 18). The productivity draw follows i.i.d. distribution $\phi(\cdot)$,¹⁵ and firms continue to be sorted by their production costs. For entry to be dynamic, there must be exit. Assume that firms die randomly at rate γ in every period. Our analysis will focus on a steady state equilibrium referred to as a free-entry equilibrium. Let α^c and α^m be the mass of firms under autarky and trade respectively. In correspondence, for fixed and endogenous labor respectively, equations (3) and (6) can be rewritten as

$$\int_0^{\kappa} \alpha \int_0^{mq(j)} c(q, j) dq \phi(j) dj = l; \quad (7)$$

$$\int_0^{\kappa} \alpha mu(q(j)) \phi(j) dj = Q. \quad (8)$$

The free-entry equilibrium conditions when the Hopenhayn entry/exit is at work are therefore characterized by (1), (2), (7), and (9) in the fixed labor case, where

$$\bar{\pi} \equiv \int_0^{\kappa} \pi(j) \phi(j) dj \leq f_e, \quad (9)$$

which says that the (unconditionally) expected profit must not exceed the entry cost. Note that $\bar{\pi}$ is independent of α . For the endogenous labor case, the equilibrium conditions are (4), (5), (8), and (9).

In a free-entry equilibrium, if there is no exogenous exit ($\gamma = 0$), then condition (9) may hold in strict inequality. This happens if trade reduces the average profitability, which results in no-entry and consequently sustains the free-entry equilibrium given that there is no exit either. The crucial question, of course, is whether trade can indeed reduce the average profitability (when $\gamma = 0$). The answer is yes. While trade does allow a firm to expand globally and capture profitable opportunities abroad, it also has two countervailing effects. For one thing, it allows other countries' firms to expand to a firm's home market and, as a result, each of these markets will become less profitable even without any new entry; that is, $f'(Q)$ decreases as a result of an increase in Q . For another, trade increases demand for resources (labor in this case) so that the opportunity cost of production increases, that is, an increase in λ . Adding up these effects together, there is no compelling force for trade to necessarily make the tradable sector more profitable to induce entry.

¹⁵As we will see, the density function ϕ is frivolous for our ensuing analysis and can be fixed to one as we did in the preceding section without any loss of generality.

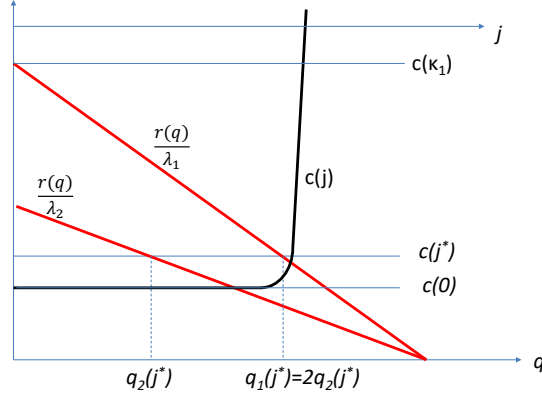


Figure 2: Trade increases most firms' labor demand if $\lambda_2 = 2\lambda_1$

Nevertheless, such ambiguity in profits is still accompanied by the unambiguity in crowding-out. In fact, exactly because each market becomes less profitable (higher λ or Q), the marginal firm, which produces zero output under autarky, must be crowded out.

To illustrate this ambiguity in profitability, consider the following scenario. Without trade, some very unproductive firms are able to survive in the economy. Their presence helps raise the price level and hence boost up the profitability of the rest of the firms that are much more productive. When the country opens to trade and these very unproductive firms are crowded out (without any new entrants) as a result, the competition among the remaining, much more productive firms will intensify significantly thanks to their similarity in productivity. This in turn drives down the prices drastically and hence the profitability of all surviving firms.

Consider trade between two symmetric countries with fixed labor. For firm j in any of the two countries, the marginal revenue is $\frac{r(q(j))}{\lambda}$ and the marginal cost is $c(j)$ (assuming constant marginal cost). Its profit is $\int_0^{q_1(j)} \left[\frac{1}{\lambda_1} r(q) - c(j) \right] dq$ before trade, and $2 \int_0^{q_2(j)} \left[\frac{1}{\lambda_2} r(q) - c(j) \right] dq = \int_0^{q_2(j)} \left[\frac{2}{\lambda_2} r(q) - 2c(j) \right] dq$ after trade. Should $\lambda_2 \geq 2\lambda_1$, then given $q_2(j) < q_1(j)$, trade must reduce this variety's profit.

Figure 2 delineates a case where the shadow price of labor has to more than double as a result of trade. In the figure, $\frac{r(q)}{\lambda_1}$ and $\frac{r(q)}{\lambda_2}$ are the two marginal revenue curves in autarky and trade respectively, with $\lambda_2 = 2\lambda_1$. For any firm j with marginal cost $c(j)$ (which is constant in q), its equilibrium output is determined from the intersection between its marginal cost and the corresponding marginal revenue. Suppose a firm j^* has a cost curve $c(j^*)$ such that it produces exactly the same amount of output, i.e., $q_1(j^*) = 2q_2(j^*)$, and therefore uses the same amount of labor.

Assume that the marginal firm under autarky is κ_1 , i.e., its marginal cost line $c(\kappa_1)$ crosses

$\frac{r(q)}{\lambda_1}$ at $q = 0$. The figure depicts a cost distribution of firms that is extremely convex, with $c'(j)$ sufficiently large for any $j \geq j^*$. Accordingly, the amount of labor saved by firms which reduce total outputs after trade must be capped by $c(\kappa_1)q_1(j^*)(\kappa_1 - j^*) \approx 0$ since κ_1 is very close to j^* due to the extremely convex cost distribution around j^* . Meanwhile, it is obvious the total output of every firm $j \in [0, j^*)$ must increase after trade: $2q_2(j) > q_1(j), \forall j \in [0, j^*)$. In other words, each of these firms will demand more labor should λ_2 merely double λ_1 . Therefore, λ must be more than doubled after trade.

Reflecting the above intuition, we derive the next proposition that highlights a sufficient condition for trade to reduce the profit of each and every operating firm (conditional on zero entry and exit).

Proposition 3 *Suppose that $\gamma = 0$. Suppose in addition that marginal cost is constant in output. Let $\kappa(m)$ be the marginal firm in trade among m symmetric countries, and let $z \in (1, m)$. Then fixing $\alpha = \alpha^c$, trade reduces every operating firm's profit if $-z\kappa'(z) > \frac{c(\kappa)}{c'(\kappa)}$ for all z .*

Proof. Unless provided, all proofs are relegated to an online appendix, the address of which is available at the end of the article.

We add the following numerical example to further illustrate the possibility.

Example 1 *Let $u(q) = aq - \frac{b}{4}q^2$ and $c(j) = c_0 + dj^x$ with $x \geq 1$. In the case of fixed labor, trade among m symmetric countries reduces the profit of every surviving firm in every country if*

$$\lambda > \frac{a}{c_0} \frac{1}{x(2x-1)}.$$

After substituting the endogenous λ , the above condition can be expressed in terms of exogenous parameters:

$$\frac{x[(x-1)(2x+1)c_0]^{\frac{x+1}{x}}}{(x+1)(2x-1)d^{\frac{1}{x}}} > \frac{lb}{ma}.$$

For any m , there exists $x(m)$ such that the above condition holds when $x > x(m)$.

In Hopenhayn dynamics, entry is determined jointly by (exogenous) exit and how trade impacts firms' profitability. If trade reduces the tradable sector's average profitability and there is no exogenous death of firms ($\gamma = 0$), there will be no net entry in equilibrium after trade, as stated in the following corollary.

Corollary 1 *Suppose that $\gamma = 0$. Suppose in addition that, fixing $\alpha = \alpha^c$, trade reduces the average profitability in the tradable sector. Then there exists a unique free-entry equilibrium where $\alpha^m = \alpha^c$.*

Notice that even though $\alpha^m = \alpha^c$ in equilibrium, trade continues to crowd out less productive firms via the mechanism highlighted in our model. Therefore, the presence of Hopenhayn entry possibility does not imply Hopenhayn entry in equilibrium, and crowding-out takes place (when countries are symmetric) even without Hopenhayn entry in equilibrium. In fact, in the scenario considered above, it is exactly the departure of less productive firms that leads to a reduction in firm profits, which in turn discourages Hopenhayn entry in equilibrium.

Now consider what happens when $\gamma > 0$. In that case, a free-entry equilibrium entails (9) to hold in equality. Recall that firm j 's profit is

$$\pi(j) = \frac{u'(q(j))}{\lambda}mq(j) - \int_0^{mq(j)} c(q, j)dq, \text{ for } j \in [0, \kappa]$$

in the fixed labor case, and

$$\pi(j) = f'(Q)u'(q(j))mq(j) - \int_0^{mq(j)} c(q, j)dq, \text{ for } j \in [0, \kappa]$$

in the endogenous labor case.

Fixing λ (or Q), an increase in m increases firm profits, and hence the average profit $\bar{\pi}$ would increase unless λ (or Q) increases and hence κ decreases. Therefore, the free-entry condition (9) implies that opening to trade (i.e., an increase in m) crowds out less productive firms:

Proposition 4 *Suppose that the exit rate is positive: $\gamma > 0$. Then there exists a unique free-entry equilibrium before and after trade. In addition, trade crowds out less productive firms ($\kappa^m < \kappa^c$).*

Proof. The uniqueness is established by the fact that $\bar{\pi}$ is decreasing in λ , and the existence is guaranteed by Property (4*). Crowding-out is explained in the text. Q.E.D.

As suggested earlier, while trade allows a firm to expand globally, it also intensifies competition in the firm's home market and increases demand for resources. With firm (exogenously) exiting, however, these last two forces are no longer sufficient to dictate firm profitability in a steady state, and this is where the Hopenhayn mechanism kicks in. Free entry/exit implies that, in a steady state, trade must not raise average profits over the entry cost, which is assumed to be invariant as in the literature.¹⁶ Consequently, each single market must become less profitable after trade in a steady state.

The result in Proposition 4 is in stark contrast to Zhelobodko et al. (2012), and Mrázová and Neary (2017). Both studies adopt the framework of Melitz (2003), but extend the preferences

¹⁶This assumption of invariant entry cost does not hold if countries are asymmetric and the entry cost requires resources such as labor, in which case the wage rate must be endogenously determined in trade.

beyond (but inclusive of) CES preferences. They show that trade may tilt firms' profitability in favor of less productive firms, which will result in crowding-in in symmetric trade. By contrast, preferences are rather general in our model, and crowding-in never takes place (in symmetric trade) regardless of the preferences other than Property (1*). The difference, it turns out, is rooted in the fixed cost of production, which is excluded from our model but is part of the staple in the existing literature. Without fixed cost, the marginal firm in our model produces zero output and, as a result, the global expansion effect of trade is minimal for such a firm: $\frac{\partial \pi(\kappa)}{\partial m} = 0$ as $q(\kappa) = 0$. Consequently, the marginal firm only suffers from the fact that trade makes each market less profitable. When the fixed cost is present as in Zhelobodko et al. (2012), and Mrázová and Neary (2017), however, $\frac{\partial \pi(\kappa)}{\partial m} > 0$ because $q(\kappa) > 0$. Therefore, depending on the preferences, the global expansion effect of trade may dominate the competition effect for the marginal firm, in which case crowding-in ensues. It is also useful to compare such mechanism with Melitz (2003) and Melitz and Ottaviano (2008). In these two studies, the marginal firm is locked by the export cost to the domestic market, and hence can only suffer from the fact that trade makes each market less profitable.¹⁷ Not being able to enjoy the global expansion effect, the marginal firm must be crowded out.

Although the Hopenhayn mechanism suggests that each market must become less profitable after trade in the steady state, the question remains as to exactly what leads to the reduced profitability. Is it a net entry of domestic and foreign firms (i.e., $\alpha^m > \alpha^c$)? Or is it the expansion of surviving incumbent firms (i.e., $\int_0^\kappa \int_0^{mq(j)} c(q, j) dq dj > \int_0^{\kappa^c} \int_0^{q(j)} c(q, j) dq dj$ for fixed labor and $\int_0^\kappa mu(q(j)) dj > \int_0^{\kappa^c} u(q(j)) dj$ for endogenous labor), in which case trade will be accompanied not by a net entry but by an exodus of firms both at home and abroad (i.e., $\alpha^m < \alpha^c$)? The next proposition provides the answer.

Proposition 5 *Suppose that the exit rate is positive: $\gamma > 0$. The free-entry equilibrium features $\alpha^m > \alpha^c$ if, conditional on $\gamma = 0$ and $\alpha = \alpha^c$, trade increases the expected firm profit $\bar{\pi}$; and the free-entry equilibrium features $\alpha^m < \alpha^c$ if, conditional on $\gamma = 0$ and $\alpha = \alpha^c$, trade reduces the expected firm profit $\bar{\pi}$.*

Proof. We prove for the case of fixed labor only. The logic for the endogenous labor case is the same and hence is omitted.

In solving the post-trade free-entry equilibrium, $q(j)$ and κ are determined by λ by conditions (1) and (2), whereas λ is determined by condition (9). This leaves α to be solved from condition (7).

Assuming that $\gamma = 0$ and fixing $\alpha = \alpha^c$ (i.e., there is neither exit nor entry), condition (9) then becomes irrelevant for solving the post-trade equilibrium, and λ is instead determined by

¹⁷Neither Zhelobodko et al. (2012) nor Mrázová and Neary (2017) entertains export cost.

condition (7). If trade increases $\bar{\pi}$ in this case, then in order for condition (9) to hold in the free-entry equilibrium when $\gamma > 0$, λ must increase. This in turn will leave condition (7) to become slack in the free-entry equilibrium unless $\alpha^m > \alpha^c$. Likewise, if trade reduces $\bar{\pi}$ conditional on $\gamma = 0$ and fixing $\alpha = \alpha^c$, in order for condition (9) to hold in the free-entry equilibrium when $\gamma > 0$, λ must decrease. Then condition (7) holds in the free-entry equilibrium only if $\alpha^m < \alpha^c$.¹⁸

Q.E.D.

Proposition 5 suggests that entry/exit mechanism *a la* Hopenhayn does not imply that less productive firms are crowded out by new entry of firms. When there are exogenous exits, the mechanism only implies that the global expansion effect of trade must be exactly offset. The global expansion effect is offset in two ways. Either each market is not “crowded” enough ($\lambda(\gamma = 0, \alpha = \alpha^c)$ or $Q(\gamma = 0, \alpha = \alpha^c)$ is not high enough for condition (9) to hold), and hence the global expansion effect must be offset with a net entry of firms: $\alpha^m > \alpha^c$. Or each market is already too “crowded” ($\lambda(\gamma = 0, \alpha = \alpha^c)$ or $Q(\gamma = 0, \alpha = \alpha^c)$ is so high that conditions (9) holds in strict inequality). In that case the global expansion effect is more than offset, and it must then be moderated by an exodus of firms ($\alpha^m < \alpha^c$) which is possible only when there are exogenous exits ($\gamma > 0$). Absent such exit, no entry will take place (as stated in Corollary 1).

Does entry/exit *a la* Hopenhayn explain the crowding-out of less productive firms? The answer is no. As we have seen, fixing the firm mass in the economy (i.e., $\gamma = 0$ and $\alpha = \alpha^c$), trade always crowds out less productive firms: $\kappa(\gamma = 0, \alpha = \alpha^c) < \kappa^c$. Per Proposition 5, if $\alpha^m > \alpha^c$, then $\kappa^m < \kappa(\gamma = 0, \alpha = \alpha^c)$, i.e., entry/exit *a la* Hopenhayn merely intensifies the crowding-out; if $\alpha^m < \alpha^c$, however, then $\kappa(\gamma = 0, \alpha = \alpha^c) < \kappa^m < \kappa^c$; that is, entry/exit *a la* Hopenhayn actually moderates the crowding-out. In fact, should we regard the equilibrium conditional on $\gamma = 0$ and $\alpha = \alpha^c$ as the short term response to trade, then trade always crowds out less productive firms in the short run. Over time at the steady state, the crowding-out either gets intensified or moderated depending on whether trade also increases or decreases the short-run average firm profitability.

3.3 Trade Effect versus Size Effect

The existing literature has equated trade, i.e., the integration of markets, to the expansion of a single market. Such an equivalence is valid in the presence of the Hopenhayn entry/exit

¹⁸ Although Proposition 5 is established when the entry cost is treated as a psychological cost, the result remains intact if entry does require labor. In that case, the free-entry condition (9) will imply that all profits are paid to labor for the entry purpose, and hence $R = l$ where R is total revenue. Since $R = \alpha \int_0^{\kappa} \frac{u'(q(j))}{\lambda} mq(j) dj$, it is clear that α is increasing in λ . Hence, Proposition 5 remains intact. The case for endogenous labor supply is even more straightforward, as labor is drawn from the non-tradable sector at a constant marginal cost, so condition (8) is not affected when labor is used on entry.

mechanism (see footnote 19). However, are they still equivalent once such a mechanism is absent? To entertain this question, we will analyze how the expansion of an autarkic country in size affects the marginal firm's operation decision. Note that while our model abstracts away from entry *à la* Hopenhayn, it does allow an inactive firm to resume operation once the underlying economic environment changes.

Consider first the endogenous labor case. The equations below present side by side the equilibrium conditions for the size effect (on the left) and trade effect (on the right), where N represents the population size of a single economy, and m represents the number of symmetric countries participating in costless trade. They differ only in the last equations, which highlight the crucial difference: trade, i.e., market integration, brings new and (stochastically) equally productive varieties, but the sheer expansion in population size does not have such an effect.¹⁹

$$f'(Q)r(q(j)) = c(Nq(j), j), \quad f'(Q)r(q(j)) = c(mq(j), j); \quad (10)$$

$$f'(Q)u'(0) = c(0, \kappa), \quad f'(Q)u'(0) = c(0, \kappa); \quad (11)$$

$$\int_0^\kappa u(q(j))dj = Q, \quad \int_0^\kappa mu(q(j))dj = Q. \quad (12)$$

Proposition 6 *Suppose a country grows in population size. In the endogenous labor case,*

- a) *if marginal cost is constant in output, then a greater population size has no impact on the number of varieties (i.e., $\kappa^N = \kappa^c$);*
- b) *if marginal cost increases with output, then a greater population size crowds in less productive firms (i.e., $\kappa^N > \kappa^c$).*

Proposition 6 highlights a stark contrast between the size effect and the trade effect when labor supply to the tradable sector is endogenous. As discussed earlier, trade brings new varieties from abroad and therefore raises each consumer's consumption composite in the tradable sector. This in turn reduces the marginal value of labor in the tradable sector: $f'(Q)$ decreases, thus crowding out less productive firms from the tradable sector. By contrast, when the population size increases in a

¹⁹ In the presence of the Hopenhayn entry/exit, condition (12) will be written as

$$\int_0^\kappa \alpha^N u(q(j))dj = Q, \quad \int_0^\kappa m\alpha^m u(q(j))dj = Q,$$

where α^N and α^m are the mass of firms in a single country in both cases. In either case, the steady-state equilibrium is solved by the above equation along with (10), (11), and (9). Since (10) and (11) solve $q(j)$ and κ for any given Q and (9) solves Q , the two free-entry equilibrium outcomes yield the same $q(j)$, κ , and Q for any $N = m$. Meanwhile, the mass of firms for the enlarged country is α^N and the mass of firms for all trading countries combined is $m\alpha^m$. Given that the two equilibrium outcomes have the same Q and $q(j)$, one can easily tell from the above equation that $\alpha^N = m\alpha^m$. That is, the mass of firms for the enlarged country equals the mass of firms for all trading countries combined. Hence the equivalence.

single economy, no new varieties come from abroad, while each existing variety's supply becomes more costly (when marginal cost increases in output). This reduces each consumer's consumption composite and thus raises the marginal value of labor in the tradable sector: $f'(Q)$ increases. As this happens, less productive firms are crowded in.

Turning now to fixed labor, we present the equilibrium conditions for the size effect (on the left) and trade effect (on the right) below. Once again, the difference lies in the last equations: trade allows firms to serve more markets without any increase in resources (labor), whereas a larger population size gives firms the luxury to serve more consumers with an enlarged labor force.

$$\frac{r(q(j))}{\lambda} = c(Nq(j), j), \quad \frac{r(q(j))}{\lambda} = c(mq(j), j); \quad (13)$$

$$\frac{u'(0)}{\lambda} = c(0, \kappa), \quad \frac{u'(0)}{\lambda} = c(0, \kappa); \quad (14)$$

$$\int_0^{\kappa} \int_0^{Nq(j)} c(q, j) dq dj = Nl, \quad \int_0^{\kappa} \int_0^{mq(j)} c(q, j) dq dj = l. \quad (15)$$

Such difference implies that, in the case of fixed labor, the size effect can be decomposed into two parts. First, an increase in population size brings more consumers to each firm, as does trade. This tends to crowd out less efficient firms, similar to the trade effect. Second, an increase in population also increases the labor supply, which tends to lower the shadow price of labor and hence crowd in less productive firms. Combining the two counterbalancing impacts, the size effect may go either way (when marginal cost increases in output).

Proposition 7 *Suppose that a country grows in population size. In the case of fixed (per capita) labor supply to the tradable sector,*

- a) *if marginal cost is constant in output, then a greater population size has no impact on the number of varieties (i.e., $\kappa^N = \kappa^c$);*
- b) *if marginal cost increases with output, then a marginal increase in population size crowds in less productive firms (i.e., $\kappa^N > \kappa^c$) if $-\frac{r(q^c(j))c'(q^c(j))}{r'(q^c(j))c(q^c(j))}$ is sufficiently large for $\forall j$, and crowd out less productive firms under the opposite condition.*

Figure 3 illustrates the two possible size effects under increasing marginal cost when population doubles. In both panels, the dark blue area indicates the amount of labor employed by firm j before the population change, and the light purple area is the additional labor by firm j after the population change *should* λ remain unchanged. In the left panel, the light purple area is smaller than the dark blue area, suggesting that, should λ remain the same, the increased labor demand from firm j as a result of population expansion is less than doubled. Evidently, if this is true for all j , then the

increased labor demand must fall short of the expanded labor supply, and hence the expansion in population must lead to a decrease in λ , thus crowding in less productive firms. The opposite is true for the right panel.

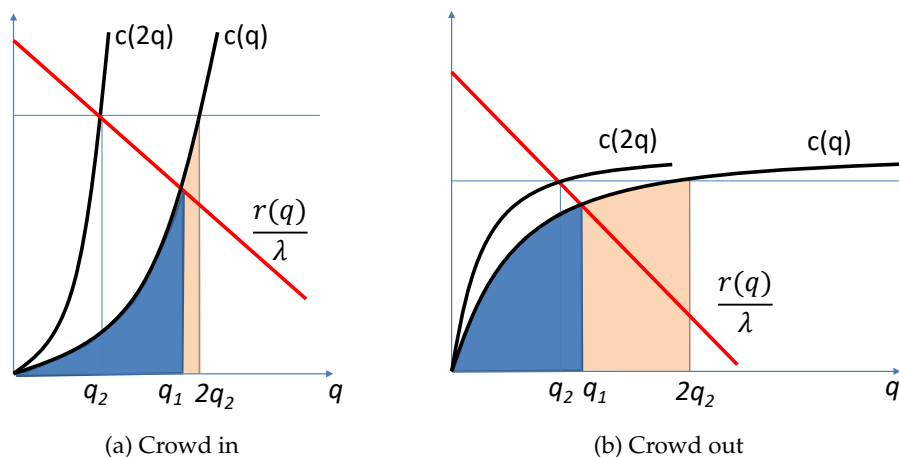


Figure 3: The size effect: fixed labor and increasing marginal cost

3.4 Source of Gains

The preceding analysis shows that, as far as our model is concerned, resource reallocation attained by trade has nothing to do with either size or entry. What, then, is the source of gains from trade with heterogeneous firms?

To answer this question, we look at how trade may open up *new* social surplus. While monopolistic firms may not exhaust all social surplus, it is nevertheless evident that there will be no trade without any new social surplus becoming available in the first place. Ultimately, it is the emergence of new social surplus that drives the competitive move of firms in their effort to capture (a share of) this new social surplus, which ultimately leads to resource reallocation across firms.

Figure 4 illustrates how trade opens up new social surplus in the context of endogenously supplied labor (the fixed labor case can be similarly depicted). The figure shows marginal cost and marginal utility as functions of consumption quantity q in the autarky equilibrium, where $f'(Q^c)u'(q)$ is the marginal value for any variety that is produced and consumed, $c(q^c(0), 0)$ is the marginal cost of the most productive firm, and $c(q^c(\kappa), \kappa)$ is the marginal cost of the least productive firm in operation. In the autarky equilibrium, $q^c(\kappa) = 0$, and a “new” variety is produced by the least productive firm at marginal cost $c(q^c(\kappa), \kappa)$ to generate $f'(Q^c)u'(0)$.

After opening to trade, suppose Q^c remains unchanged but labor is reallocated from the least productive firm to the most productive firm. The former will stop producing that “new” variety,

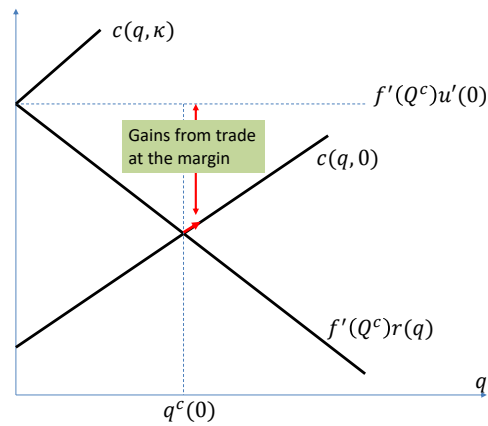


Figure 4: The source of gains from trade

while the latter can increase its production at a slightly higher marginal cost (as indicated by the short red arrow). The extra output of the most productive firm can then be used to trade for some variety “new” from abroad, which also generates $f'(Q^c)u'(0)$. Since $f'(Q^c)u'(0) > f'(Q^c)u'(q^c(0)) = c(q^c(0), 0)$, trade brings unambiguous, discrete welfare gain (as shown by the vertical red arrows). This is the fundamental force that brings gains from trade when firms are heterogeneous.

Such force does not work for the marginal firm, whose production is already generating a marginal utility of $f'(Q^c)u'(0)$ under autarky. Although extending production can bring some new variety through trade, the marginal utility $f'(Q^c)u'(0)$ thus generated never dominates the (weakly) increased marginal cost.

Therefore, the gain in marginal benefit must dominate the slight increase in marginal cost. While this is only a marginal effect (i.e., allowing individual firms to adjust production but fixing the industry-wide Q or λ), since the very increase in welfare will change Q , giving rise to further reallocation of resources that will unfold until an equilibrium is reached, it should be evident that without such a marginal effect to begin with, there would be no resource reallocation from trade. In plain language, then, the driving force behind the gains from trade with heterogeneous firms is:

Without trade, anything new must be produced domestically, which is increasingly costly. With trade, a country can trade for something new by producing what it is best at. Since the marginal gain of obtaining that new thing from abroad outweighs the marginal cost of expanding the production of the most productive firm, hence the trade.

4 Trade Among Asymmetric Countries

The analysis thus far deals with trade among symmetric countries, in which case less productive firms are always crowded out. This no longer holds when countries are asymmetric, as we will show next.

After re-characterizing the equilibrium conditions for the fixed and endogenous labor cases, our analysis of trade among asymmetric countries will be carried out in four installments. Section 4.1 shows that there will always be some countries where the less productive firms specialize in exports. Section 4.2 analyzes in what kind of countries such a phenomenon will arise. Section 4.3 shows that a firm's export intensity depends not only on the standing of the firm in its home country in terms of productivity, but also on the standing of the home country in the world in terms of its size and/or overall productivity. Finally, Section 4.4 demonstrates the possibility for trade to crowd in less productive firms.

When countries are asymmetric, wage rates can no longer be normalized. Let w^x denote the wage rate in country x . For any variable that is country-specific, such as w , λ , Q , $c(q(j), j)$, N (population size), we use superscript to denote that country's identity. For a variable that is related to trade between two countries, such as q , κ , p , we use subscript to indicate the origin country, and superscript to indicate the destination country. Following Chaney (2008), we assume that each firm can practice price discrimination across different countries and hence price its variety differently. Together with Property (2*), this assumption implies that in each market, every firm faces a downward sloping marginal revenue curve.

As before, we entertain both fixed and endogenous labor supply. In the fixed labor case, consumers in destination country x chooses $q_h^x(j)$ for product j from origin country h to maximize

$$f\left(\sum_{h=1}^m \int_0^\infty u(q_h^x(j))dj\right),$$

s.t.

$$w^x l^x + \int_0^\infty \pi^x(j) dj = \sum_{h=1}^m \int_0^\infty p_h^x(j) q_h^x(j) dj.$$

In equilibrium, should there be export from country y to country x , the following conditions hold:

$$\frac{w^x}{\lambda^x} r(q_y^x(j)) = w^y c_y \left(\sum_{h=1}^m N^h q_y^h(j), j \right), \text{ for } j \in [0, \kappa_y^x], \quad (16)$$

$$\frac{w^x}{\lambda^x} u'(0) = w^y c_y \left(\sum_{h \neq x}^m N^h q_y^h(\kappa_y^x), \kappa_y^x \right). \quad (17)$$

Equation (16) equates the marginal cost of production by firm j of the origin country y to the marginal revenue it earns in the destination country x , factored by the wage rates of these two countries and the shadow price of labor in the destination country. When it is not constant, the marginal cost depends on j 's global production, $\sum_{h=1}^m N^h q_y^h(j)$. Equation (17) applies (16) to the marginal firm exporting from y to x , i.e., κ_y^x . If countries are symmetric, the sale of this variety in each and every country is zero by definition. However, when countries are asymmetric, the marginal firm exporting from y to x may well produce and sell a positive amount in countries other than x ; accordingly its total production is $\sum_{h \neq x}^m N^h q_y^h(\kappa_y^x)$.

In the endogenous labor case, consumers in destination country x chooses l^x and $q_h^x(j)$ for product j from origin country h to maximize:

$$f\left(\sum_{h=1}^m \int_0^{\infty} u(q_h^x(j)) dj\right) - l^x,$$

s.t.

$$w^x l^x + \int_0^{\infty} \pi^x(j) dj = \sum_{h=1}^m \int_0^{\infty} p_h^x(j) q_h^x(j) dj.$$

The equilibrium conditions are:

$$w^x f'(Q^x) r(q_y^x(j)) = w^y c_y\left(\sum_{h=1}^m N^h q_y^h(j), j\right) \text{ for } i \in [0, \kappa_y^x], \quad (18)$$

$$w^x f'(Q^x) u'(0) = w^y c_y\left(\sum_{h \neq x}^m N^h q_y^h(\kappa_y^x), \kappa_y^x\right). \quad (19)$$

We shall consider two types of asymmetry: one in terms of productivity and the other in terms of population size. Before examining how asymmetric equilibrium may arise endogenously from these exogenous asymmetry, let's first look at the implications of an asymmetric equilibrium on individual firms' export behavior.

4.1 Less Productive Firms (in Some Countries) Specialize in Exports

Equations (17) and (19) characterize the marginal firm exporting from y to x in the fixed and endogenous labor cases respectively. A close look at these equations immediately reveals the following:

Proposition 8 *Suppose that, among the m countries, there exists country x with the largest aggregator, that is, $w^x f'(Q^x) = \arg \max_h w^h f'(Q^h)$ for endogenous labor, and $\frac{w^x}{\lambda^x} = \arg \max_h \frac{w^h}{\lambda^h}$ for fixed labor. Then*

- a) Less productive firms in country y will specialize in exports (i.e., they do not sell in domestic market) if and only if the aggregator of y is smaller than that of x .
- b) In every country, firms that specialize in exports, if any, are less productive than firms that serve the domestic market.
- c) In countries with the largest aggregator, no firms specialize in export, and less productive firms serve their domestic market.

Proof. The proof is straightforward and is thus omitted.

Proposition 8 can be illustrated in Figures 5 and 6, which assume endogenous labor supply and constant marginal cost in output. Figure 5 considers trade between two countries. In equilibrium, one country (country x) is richer than the other (country y) in the sense that $w^x f'(Q^x) > w^y f'(Q^y)$. The left panel of Figure 5 shows that in the poor country, the less productive firms specialize in export, whereas the right panel of Figure 5 shows that in the rich country, the less productive firms serve the domestic market only.

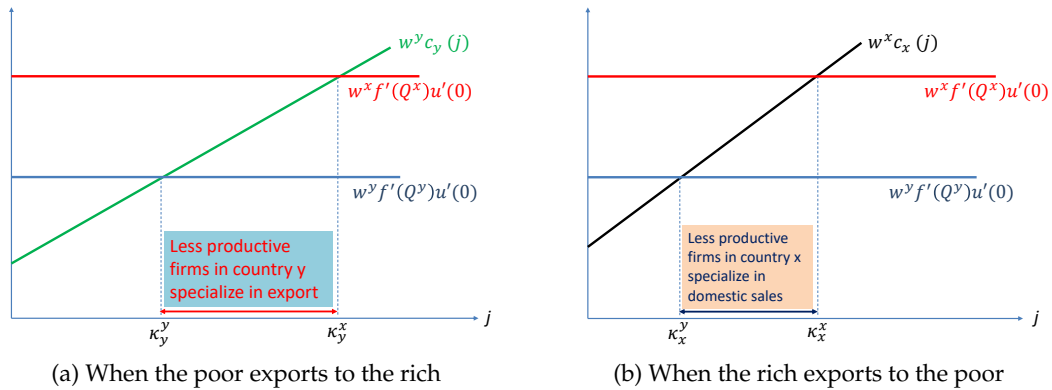


Figure 5: Trade between two asymmetric countries

Figure 6 considers trade among three countries, x, y, z , where country z is the medium income country in equilibrium: $w^x f'(Q^x) > w^z f'(Q^z) > w^y f'(Q^y)$. It illustrates how origin country $h \in \{x, y, z\}$ serves any of the three destination countries. Evidently, the less productive firms in country h will not specialize in exports only if $h = x$.

The intuition for Proposition 8 is straightforward. Demand from a destination country is factored by the country's aggregator, that is, $w f'(Q)$ in the endogenous labor case and $\frac{w}{\lambda}$ in the fixed labor case. An origin country faces demands from different destination countries (including itself) for its products. Provided that these aggregators differ across countries, there will be different thresholds for exports to various destination countries. When foreign demand is stronger than

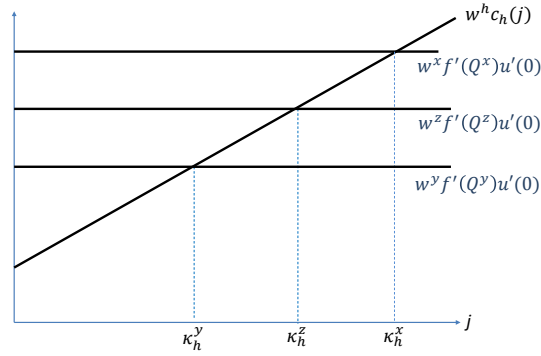


Figure 6: Trade among three asymmetric countries

domestic demand, the least productive varieties are affordable to foreign buyers but not domestic consumers. These firms will therefore specialize in exports.

The presence of a small trade cost is unlikely to perturb Proposition 8. This is because goods that are not affordable in some countries cannot become affordable due to the presence of trade cost. In other words, the presence of a small trade cost may reduce export by the less productive firms, but is unlikely to turn those firms to serve their domestic markets instead, as the presence of trade cost cannot make goods more affordable to a domestic market that otherwise cannot afford them.

The possibility suggested by Proposition 8 cannot arise when countries are symmetric, however. This is because when countries are symmetric, the aggregators of all countries are equalized in equilibrium. As a result, a variety is exported only if it is also sold in its domestic market; hence there can never be any firm specialize in exports. In correspondence, the existing literature has emphasized the opposite pattern: in the presence of trade cost, the less productive firms specializing in domestic sales instead.

Chaney (2008) and Melitz and Ottaviano (2008) have also examined asymmetric trade. Their analyses assume that all countries share the same constant returns to scale technology on a tradable numeraire and focus on the case where that numeraire good is produced by every country in a trade equilibrium. Accordingly, the asymmetry as we highlight here can help enrich their analyses.

4.2 The Pecking Order of Trade

Proposition 8 hinges on asymmetry in aggregators that are endogenously determined. We now establish the existence and patterns of such asymmetry from model primitives and draw further implications. Two kinds of primitives are considered which can lead to the asymmetry in aggregators: one is the difference among countries in their overall productivity, and the other is

the difference in country sizes.

Consider first the asymmetry in productivity. A country x is said to dominate another country y in productivity if, after sorting, the marginal cost of firm j in x is lower than the marginal cost of the firm in y with the same rank: $c_x(q, j) < c_y(q, j)$ for any given q, j . Note that this dominance in productivity is different from absolute advantage in international trade because here firms are producing distinctive varieties.

Proposition 9 *Suppose that among m countries there exist at least two countries with one dominating the other in productivity. Then in a trade equilibrium involving these m countries, there exist countries x, y such that $w^x f'(Q^x) \neq w^y f'(Q^y)$ in the case of endogenous labor, and $\frac{w^x}{\lambda^x} \neq \frac{w^y}{\lambda^y}$ in the case of fixed labor.*

Proposition 9 says that, as long as one country in the world dominates another country in productivity, the aggregators cannot be identical across all countries in the world. The reason is simple. Had all countries have the same aggregator, their spending as destination must be the same given that they face the same set of varieties supplied in the global market. However, productivity difference implies that countries as origin must have different earnings. As a result, trade cannot be balanced in every country.

The next question is, which country will have a larger aggregator? Common sense suggests that countries with higher productivity must end up richer, thus having larger aggregators and hence stronger purchasing power. This is indeed the case, as the next proposition states.

Proposition 10 *Suppose that a subset of m countries can be ranked in terms of productivity dominance. Then in a trade equilibrium involving these m countries, $\frac{w^x}{\lambda^x} > \frac{w^y}{\lambda^y}$ (in the case of fixed labor) or $w^x f'(Q^x) > w^y f'(Q^y)$ (in the case of endogenous labor) if country x dominates country y in productivity.*

The intuition for Proposition 10 can be explained as follows. Given that country x dominates country y in productivity, firms in country x must generate more earnings than firms in country y . Therefore, in equilibrium, country x must have higher spending than country y , which is possible only if country x has a larger aggregator.

As more productive countries have larger aggregators, they will spend more. To spend more, they must be more competitive internationally, as the next proposition states.

Proposition 11 *Suppose that a subset of m countries can be ranked in terms of productivity dominance. Then in a trade equilibrium involving these m countries, $w^x c_x(q, j) < w^y c_y(q, j)$ for any given q and j if country x dominates country y in productivity.*

Proof. Otherwise, more productive countries will earn less revenue, contradicting the fact that they will spend more. Q.E.D.

With the equilibrium costs of production thus ranked, we can then determine what we refer to as the pecking order of trade in the case of productivity asymmetry.

Proposition 12 *Suppose that a subset of m countries can be ranked in terms of productivity dominance. Then the following patterns hold in a trade equilibrium involving these m countries.*

- a) *If a firm in country y exports to country x , then it also exports to countries that dominate country x in productivity.*
- b) *If less productive firms in country y are specialized in exports, then less productive firms in countries that are dominated by y in productivity are also specialized in exports.*
- c) *If country y specializes in exports (i.e. no firm in country y serves its domestic market), then all countries that are dominated by country y in productivity must also specialize in exports.*
- d) *If country y specializes in exports, then there is no trade between country y and any of the countries (weakly) dominated by y in productivity.*
- e) *If labor is endogenously supplied, $w^x > w^y$ if country x dominates country y in productivity.*

Another source of asymmetry is different population sizes across countries. The next proposition establishes that a larger country will have a lower wage rate and a smaller aggregator as compared to smaller countries.

Proposition 13 *Suppose that there are m countries with different sizes. Then the following patterns hold in trade:*

- a) *$w^x < w^y$ if and only if country x is larger than country y , i.e., $N^x > N^y$; and*
- b) *$\frac{w^x}{\lambda^x} < \frac{w^y}{\lambda^y}$ (in the case of fixed labor) and $w^x f'(Q^x) < w^y f'(Q^y)$ (in the case of endogenous labor) if and only if country x is larger than country y , i.e., $N^x > N^y$.*

The intuition behind Proposition 13 is as follows. A larger country supplies more labor to the global market. For the world to absorb this larger supply, its marginal costs must be lower so that its firms will indeed sell more and hence make use of more labor resources at home. This in turn requires that the labor of a larger country must be cheaper.

While the cheaper labor makes firms in large countries more competitive, the total income earned in these countries increases less than proportional to their larger population size. This is because, in the presence of increasing marginal cost and diminishing marginal revenue (Property (2*)), doubling the population size and hence the labor size will not double a nation's earning. In

accordance, the total spending in larger countries has to increase less than proportional to their larger population size. This implies that, with a larger population size, per-capita spending must be smaller in a larger country than in smaller countries, which is possible only if the aggregator in the larger country is smaller.

With equilibrium wage rates and aggregators thus ranked, we can once again determine the pecking order of trade in the case of asymmetry in country size.

Proposition 14 *Suppose that there are m countries of different sizes. Then the following patterns hold in trade.*

- a) *If a firm in country x exports to country y , then it also exports to countries that are smaller than country y .*
- b) *If less productive firms in country y specialize in exports, then less productive firms in countries that are larger than y also specialize in exports.*
- c) *If country y specializes in exports, then all countries that are larger than country y must also specialize in exports.*
- d) *If country y specializes in exports, then there is no trade between y and any country that is (weakly) larger than y .*

Proof. The proof is similar to that of Proposition 12 and hence is omitted. Q.E.D.

4.3 Export Intensity and Firm Productivity

When countries are symmetric, export share is constant across firms because every foreign market is identical to the domestic market, so the export share is always $\frac{m-1}{m}$ regardless of each variety's productivity. Clearly this does not reflect what takes place in reality, where export share tends to vary across firms. Country asymmetry can help address variations in export share across firms with heterogeneous productivity. Roughly speaking, country asymmetry leads to asymmetric demand across different markets, and firms of different productivity will have different ability to access these asymmetric demand, thus leading to variations in export shares.

To simplify the discussion, we will assume in this subsection that marginal cost of production is constant in output. Define κ_h as the least productive firm in country h beyond which there will be no production in a trade equilibrium: $\sum_{x \neq h} q_h^x(i) + q_h^h(i) = 0$ for all $i > \kappa_h$. For $i < \kappa_h$, let $e_h(i)$

denote the share of exports in total output of firm i in origin country h , and $d_h(i)$ the corresponding domestic output share:

$$e_h(i) = \frac{\sum_{x \neq h} q_h^x(i)}{\sum_{x \neq h} q_h^x(i) + q_h^h(i)} = 1 - d_h(i).$$

Differentiating $d_h(i)$ with respect to i , and making use of the assumption of constant marginal cost, we arrive at the following lemma, which offers a necessary and sufficient condition for more productive firms (smaller i) to export more:

Lemma 1 *Suppose that marginal cost of production is constant in output. Then $\frac{\partial d_h(i)}{\partial i} > 0$ if and only if:*

$$\sum_x q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right] > 0. \quad (20)$$

Recall that a country has a larger aggregator than another country if the former dominates in productivity or has a smaller size. For any origin country, a destination country with a larger aggregator represents a stronger demand and hence brings a larger sale, q , to any exporting firm in the original country. Therefore, if preferences are such that $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q , we can observe the following pattern:

Proposition 15 *Suppose that marginal cost of production is constant in output, that all m countries can be strictly ranked by their aggregators in an ascending order of m , and that $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q . Then*

- a) $e_m(i)$ decreases in i for $i \leq \kappa_m^{m-1}$ and $e_m(i) = 0$ for all $i \in (\kappa_m^{m-1}, \kappa_m^m]$;
- b) For country $h < m$, there exists $\kappa(h) \in [0, \kappa_h^{h-1}]$ such that $e_h(i)$ increases in i for $i \in [\kappa(h), \kappa_h^h]$ and $e_h(i) = 1$ for all $i \in (\kappa_h^h, \kappa_h^m]$.

To understand the intuition behind Proposition 15, consider trade between two countries, one richer (i.e., its aggregator is larger) than the other. In the rich country, every variety's export is smaller than its domestic sales. As productivity decreases, the export and domestic sales both decrease. The condition of $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increasing in q implies that the proportional decrease in export is greater than the proportional decrease in domestic sales, so the export share is smaller for a less productive variety. In fact, the least productive firms specialize in domestic sales and therefore their export share is zero. The opposite is true for the poor country.

Indeed, a simple corollary can be drawn from Proposition 15 for the case where all m countries are divided into two groups, with countries being homogeneous in each group.

Corollary 2 *Suppose that marginal cost of production is constant in output and that $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q .*

- a) *In the group with the larger aggregator, the export share weakly increases in firm productivity.*
- b) *In the group with the smaller aggregator, the export share weakly decreases in firm productivity.*

Proof. This follows directly from Proposition 15.

Proposition 15 and Corollary 2 suggest that the prediction that more productive firms export more is likely to be relevant only for rich countries. For very poor countries, the opposite can be true. For countries in between these two extremes, the relation can be more complex, with more productive firms exporting less for at least some firms. The bottom line is: the pattern of export intensity depends on two factors: firm i 's standing within its home country, and the home country's standing in the global economy. One cannot draw a pattern of export intensity across board without considering these two factors simultaneously.

Finally, we offer a class of preferences that feature increasing $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$.

Corollary 3 *Suppose that preferences features constant absolute risk aversion (CARA) and that Property (2*) holds. Then $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q .*

4.4 Crowding in Less Productive Firms

When countries are symmetric, trade always crowds out less productive firms, as established earlier. When countries are asymmetric, however, trade can in fact bring into action firms that have otherwise chosen to shut down under autarky.

Let us denote by κ_h^c the productivity threshold in country h under autarky. Trade will crowd out marginal firms in country h if and only if the productivity threshold for exporting to every destination country falls short of κ_h^c , that is: $\kappa_h^x < \kappa_h^c$ for all x .

We begin with the next lemma, which says that trade always raises the productivity threshold on the domestic market.

Lemma 2 $\kappa_h^h < \kappa_h^c$ for all h .

The reason for this lemma can be understood as follows. Regardless of country asymmetry, for any given country, trade always bids up the shadow price of labor (i.e., λ rises) in the case of fixed labor. As a result, the threshold firm serving the domestic market must be more productive to survive in trade. In the case of endogenous labor, because trade allows consumers in any given

country to consume more (i.e., Q increases), the marginal value of its own domestic varieties must fall as well, once again forcing less productive firms to quit from operating in the domestic market.

Note that Lemma 2 corresponds to, and in fact implies, Propositions 1 and 2 when countries are symmetric. That is, when countries are symmetric, firms that serve the domestic market also serve the export market and vice versa: $\kappa_h^y = \kappa_h^h$ for all y . Therefore, less productive firms must be crowded out when countries are symmetric.

When countries are asymmetric, Lemma 2 has two additional important implications. First, should some firms be crowded in as a result of trade, these firms must specialize in export. This is because, according to Lemma 2, firms that are unable to reap any profits under autarky will not be able to reap any profits on the domestic market in trade. Therefore, should they be crowded in as a result of trade, they must be serving the export market.

Corollary 4 *Suppose that crowding-in takes place in some countries, then these crowded-in firms must be specialized in export.*

Proof. This follows directly from Lemma 2.

Q.E.D.

Second, crowding-in never takes place for the richest countries, i.e., countries with the largest post-trade aggregator in equilibrium. Among all destination countries, the country with the largest post-trade aggregator presents the highest productivity threshold for its own products. Therefore, for such a country, trade crowds out marginal firms: $\kappa_h^h = \max\{\kappa_h^x\} < \kappa_h^c$.

Corollary 5 *Trade crowds out less productive firms in countries with the largest post-trade aggregator.*

Proof. The proof is straightforward and is thus omitted.

Corollary 5 in turn suggests that crowding-in, should it take place in some countries, never takes place without crowding-out occurring in other countries at the same time. This particular feature of any potential crowding-in must therefore be understood in conjunction with Corollary 4: Without trading partners where crowding-out is taking place, it is impossible for countries to materialize crowding-in simply by trading among themselves. As reasoned above, without welfare gains that are attained through crowding-out and shared across countries via certain terms of trade, it will never be profitable to re-introduce into trade the less productive firms that would have been unprofitable in autarky.

In addition, we can show that crowding-in never takes place when countries are of the same size.

Proposition 16 *Suppose that labor is in fixed supply and countries are of the same size. Then trade among countries crowds out less productive firms even when they are asymmetric in productivity.*

The intuition for Proposition 16 is as follows. According to Lemma 2, trade draws in less productive firms only if some foreign demand in trade is larger than the domestic demand under autarky (that is, the post-trade aggregator of a foreign market is larger than the pre-trade aggregator of the domestic market). If that were the case, however, each intra-margin firm would also sell more output in the corresponding foreign market than it does in its domestic market in autarky. It means that the total output in trade must exceed that in autarky, making it impossible to clear the labor market if all countries have the same size.

We have shown earlier that, for countries that do not boast the largest aggregator in trade, marginal firms will specialize in exports (Proposition 8). Whether these firms are more or less productive than the marginal firms in autarky depends on how post-trade aggregators in foreign markets compare with the pre-trade aggregator in the domestic market. As we will elaborate below, there exists some compelling reasons for the former to rise above the latter, thus crowding in less productive firms rather than driving them out.

We begin with asymmetry in productivity. Consider the following numerical example.

Example 2 *Let $u(q) = q - \frac{q^2}{4}$ and $f(Q) = \ln(Q+1)$. There are two countries, x and y , each with endogenous labor supply. The marginal cost of production is $c_x(j) = \frac{1}{10}(1+j)$ for country x and $c_y(j) = \frac{\beta}{10}(1+j)$ for country y , where $\beta \geq 1$. Note that we have relaxed Property (4*) in this numerical example. Figure 7 shows the productivity thresholds in each country as a function of the productivity gap, β .*

In Figure 7, the vertical axis represents the productivity cutoffs, whereas the horizontal axis represents β , the productivity gap between country y and country x . The larger is β , the less productive is country y as compared to country x . Under autarky, the threshold in the benchmark country x , κ_x^c , is constant, whereas the threshold in country y , κ_y^c , decreases in β .

Echoing Lemma 2, for any given β , trade lowers the productivity cutoffs in both countries' domestic markets: $\kappa_x^x \leq \kappa_x^c$ and $\kappa_y^y \leq \kappa_y^c$. As country y 's productivity gap expands, the benchmark (more advanced) country x becomes increasingly self-sufficient under trade: κ_x^x increases and approaches κ_x^c , while κ_x^y decreases. In contrast, the backward country y becomes increasingly engaged in trade; and eventually when the country becomes sufficiently backward, its tradable sector is completely specialized in export: $\kappa_y^y = 0$.

As Corollary 5 suggests, marginal firms are crowded out in the benchmark (more advanced) country x : $\kappa_x^c > \max\{\kappa_x^x, \kappa_x^y\}$. For the backward country y , $\kappa_y^x > \kappa_y^y$ in trade, suggesting that less productive firms are specialized in trade as implied by Propositions 8 and 10. More interestingly,

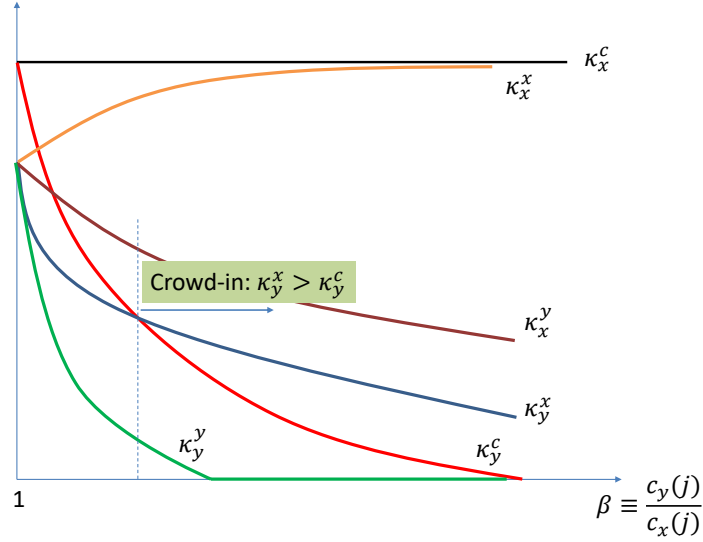


Figure 7: Crowding-in: endogenous labor with asymmetry in productivity

there exists a threshold β (highlighted by the vertical dotted line), below which crowding-out takes place: $\kappa_y^c > \max\{\kappa_y^x, \kappa_y^y\}$, and above which crowding-in takes place: $\kappa_y^c < \kappa_y^x$. Moreover, all those firms that are crowded in (i.e., for $\kappa \in (\kappa_y^c, \kappa_y^x]$ at β beyond the threshold) specialize in exports, as $\kappa_y^c > \kappa_y^y$ (reflecting Lemma 2) implies $\kappa_y^x > \kappa_y^y$.

Furthermore, when the productivity gap β becomes sufficiently large beyond the threshold, not only are marginal firms crowded into the tradable sector, but the entire tradable sector becomes specialized in exports ($\kappa_y^y = 0$ and $\kappa_y^x > \kappa_y^c$). Finally, when β becomes even larger, the entire tradable sector is out of business under autarky ($\kappa_y^c = 0$) and it takes trade to bring the sector back to life ($\kappa_y^x > 0$).

To gather intuition for the crowding-in, we turn to Figure 8, where country x dominates country y in productivity. The horizontal axis represents output of any given firm from either country, whereas the vertical axis represents the marginal values and the marginal costs. In autarky, the most productive firm in country x produces $q_x^c(0)$, determined by the intersection of its marginal cost $c_x(q, 0)$ and its marginal revenue $f'(Q^x)r(q)$. The tradable sector of country y is empty in autarky: the marginal cost of the country's most productive firm equals the marginal revenue $c_y(q, 0) = f'(Q^y = 0)r(q)$ at $q = 0$ ($Q^y = 0$ because the tradable sector is empty). Note that Property (4*) is relaxed for this scenario to take place, as $f'(0)$ is finite.

Now, suppose that the two countries open for trade. In country y , starting to produce in the tradable sector must generate a loss in social welfare *domestically*: Recall that $f'(Q)$ is decreasing in Q , so should there be trade, $f'(Q^y > 0)$ must fall further below $c_y(q, 0)$ for all q .

However, at the margin, the welfare loss in country y must be outweighed by the welfare gain

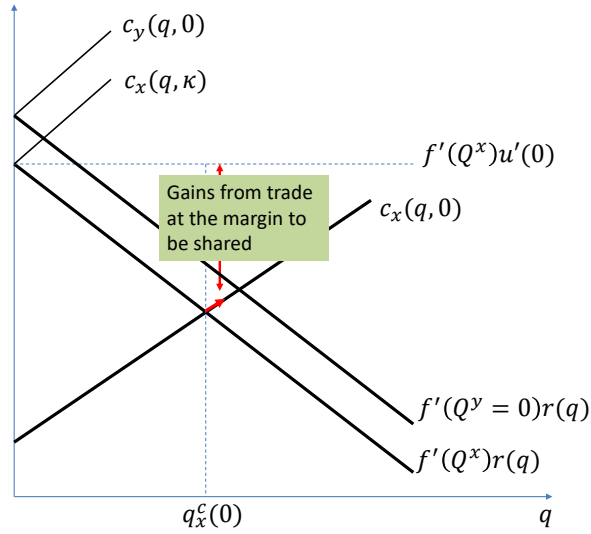


Figure 8: Crowding-in and gains from trade

in country x . To see this, let the most productive firm in country x marginally expand its production beyond $q_x^c(0)$. There will be a rise in production cost as shown in Figure 8 by the small arrow along $c_x(q, 0)$. Yet, trade allows country x to consume some new variety from country y , of which its marginal utility is $f'(Q^x)u'(0)$. The vertical double-headed line indicates the marginal gain in social welfare in country x , which must be sizeable since $f'(Q^x)u'(0) > f'(Q^x)u'(q_x^c(0)) = c_x(q_x^c(0), 0)$. Meanwhile, because $c_y(q, 0) = f'(Q^y = 0)r(q)$ at $q = 0$, the welfare loss in country y as a result of the marginal expansion in trade must be close to zero.

Since the marginal welfare gain attainable in country x outweighs the marginal welfare loss in country y , there will be a net gain from trade to be tapped. This net gain is to be shared by the two countries in order for them to engage in voluntary exchange, and the endogenous terms of trade $\frac{w^x}{w^y}$ will serve the role of dividing the gain. As a result, once trade is possible, the tradable sector of country y will come to life: trade will crowd in marginal firms, which were not viable in autarky.

The next proposition formalizes and extends this intuition:

Proposition 17 *Suppose that $f'(\cdot)$ is bounded from above and labor is endogenously supplied to the tradable sector. Suppose in addition that among m countries in trade, there exists a country y that is dominated by another country x in productivity so that $c_y(q, j) = \beta c_x(q, j)$ for any $q, j, \beta > 1$ and $c_y(0, 0) < f'(0)u'(0)$. Then there exists $\beta_{-1}, \beta_0, \beta_1, \beta_2$ with $\beta_{-1} < \beta_0 < \beta_1 < \beta_2$ such that*

- a) for $\beta \in [\beta_1, \beta_2)$, trade crowds in the entire tradable sector in country y to specialize in export;
- b) for $\beta \in [\beta_0, \beta_1)$, less productive firms in the tradable sector of country y are specialized in export and, among them, even less productive ones are crowded in as a result of trade;

c) for $\beta \in (\beta_{-1}, \beta_0)$, no crowding-in takes place but less productive firms in the tradable sector of country y are specialized in export.

Proposition 17 says that, when the marginal value of the tradable sector is bounded, less productive firms of a country that is dominated in productivity by another country with an active tradable sector will be crowded in when the productivity gap between these two countries becomes large enough.

In addition, Proposition 17 highlights a pattern of how the tradable sector in the productivity-dominated country responds to the country's development that closes its gap with more advanced nations. It says that, as the backward country closes in on the more advanced country, trade will bring its tradable sector to life, first to be specialized in serving the export market of advanced nations (see Proposition 12) before it begins to serve its domestic market. This pattern corresponds to what was shown in Figure 7.

Proposition 17 is established after Property (4*) is relaxed. One natural question is whether crowding-in may take place even under Property (4*). The answer is affirmative for a large class of, *albeit* not all, preferences:²⁰

Proposition 18 *Suppose that labor is endogenously supplied to the tradable sector. There exists a large class of preferences satisfying Property (4*) such that, when two groups of countries x and y trade among each other, with one group dominating the other in productivity so that $c_y(q, j) = \beta c_x(q, j)$ for any q, j , and $\beta > 1$, crowding-in takes place in the dominated countries when the productivity gap β becomes sufficiently large.*

The analysis above deals with crowding-in that takes place when countries differ in their overall productivity. Next, we turn to how asymmetry in size may contribute to crowding-in. Once again, we begin with a numerical example.

Example 3 *Let $u(q) = q - \frac{q^2}{4}$ and $c(j) = 1 + j$. There are two countries, x and y , with population sizes $N^x \geq N^y$ and per capita labor endowment $l = 1$ in both countries. Figure 9 shows the productivity cutoffs in each country as a function of the relative population size.*

As shown in Figure 9, the productivity thresholds in the two countries are the same under autarky regardless of their size difference, $\kappa_x^c = \kappa_y^c$, given the fact that marginal cost of all firms are constant in output (see Proposition 7).

²⁰When $f(\cdot)$ is a log function, we can show, using numerical examples, that crowding-in does not take place for a wide range of β .

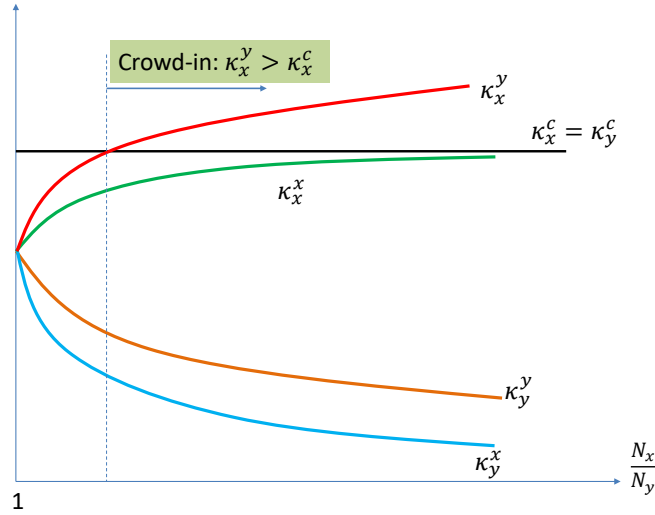


Figure 9: Crowding-in: fixed labor with asymmetry in size

When the two countries open for trade, trade crowds out marginal firms in both countries: $\kappa_y^x = \kappa_y^y = \kappa_x^x = \kappa_x^y < \kappa_x^c = \kappa_y^c$ if their population sizes are the same (on the vertical axis with $\frac{N_x}{N_y} = 1$) (see Lemma 2 and Corollary 5).

As the size differential increases beyond a threshold (represented by the vertical dotted line), crowding-in occurs in the larger country x : $\kappa_x^y > \kappa_x^c$. Meanwhile, crowding-in never takes place in the smaller country y : $\kappa_y^c > \max\{\kappa_y^x, \kappa_y^y\}$, which echoes Corollary 5 as the smaller country y has a larger aggregator as per Proposition 13.

As the size differential increases, the smaller country y becomes increasingly specialized in its most productive varieties, shunning less productive firms from both domestic and export markets, thus causing both κ_y^y and κ_y^x to decline. This phenomenon contrasts interestingly with the large country, which becomes increasingly diversified in its production portfolio, with increasingly less productive firms brought into production.

To understand the mechanism behind the crowding-in when countries differ in size, let's consider two groups of countries, large and small. In a small country, per capita spending increases with the country's aggregator. More spending by small countries means a lower productivity threshold in large countries. As a result, should trade crowd out less productive firms in large countries, the per capita spending in a small country has to be capped from above. Meanwhile, the per capita earning in a small country corresponds to the relative size of the large countries, as any particular variety from a small country must serve every consumer in a large country. If the large countries become sufficiently large, the per capita earning in a small country must grow beyond its per capita spending had the marginal firms been crowded out in large countries. As a

result, trade cannot be balanced unless less productive firms are crowded in in the large countries.

To put it simply, when the population size of the large countries is sufficiently large, the need for a small country to serve every consumer in the large countries will raise the small country's purchasing power (represented by the country's aggregator) so much that marginal firms in the large country have to be crowded in to balance the trade. Our last proposition summarizes this intuition.

Proposition 19 *Suppose that marginal cost is constant in output. Suppose further that there are m_L large country with population size $N > 1$, and m_S small countries with population size normalized to one. All countries have identical distribution of productivity. For any given (m_L, m_S) , there exists $\hat{N}(m_L, m_S)$ such that trade crowds in less productive firms in large countries if $N > \hat{N}(m_L, m_S)$.*

5 Conclusion

By developing a rather simple and tractable model, we are able to shed new lights to the insights established in the literature about trade with heterogeneous firms. While reaffirming the existing findings in a symmetric trade setting, our model not only helps crystalize the driving force behind these findings but, more importantly, highlights the limitation of these findings in an asymmetric trade environment. Our rich set of predictions highlights a stylized pattern of trade and firm behaviours that reflects not only the standing of firms within a given nation as the existing literature has typically focused on, but also the standing of that nation in the world. This pattern of trade and firm behaviours echoes the diverse empirical findings that sometimes contradict the existing theoretical insights; and it suggests that empirical findings for trade with heterogeneous firms could be country specific, and as a country develops itself along the world ladder, time specific too.

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Online Appendix

All proofs, unless already provided, are available at

<https://www.dropbox.com/s/u3zn1bz10qbmm9q/model-03-3-proofs.pdf?dl=0>

Online Appendix for “Symmetric and Asymmetric Trade with Heterogeneous Firms”

Proof of Proposition 3

In fixed labor, for firm j , the demand is $p(q(j)) = \frac{1}{\lambda}u'(q(j))$, then its profit is $\pi(j) = mq(j)p(j) - mq(j)c(j) = mq(j)\frac{1}{\lambda}u'(q(j)) - mq(j)c(j)$. Differentiating $\pi(j)$ with respect to m and using Envelope Theorem, we have $\pi_m(j) = q(j)u'(q(j))\frac{\lambda - m\lambda_m}{\lambda^2} - c(j)q(j)$, where the subscript m denotes the partial derivative with respect to m . A sufficient condition for firm j 's profit to decline is therefore $\lambda < m\lambda_m$. Since $u'(0) = \lambda c(\kappa)$, hence $\lambda_m c(\kappa) + \lambda c'(\kappa)\kappa_m = 0$, and $\frac{\lambda}{\lambda_m} = -\frac{c(\kappa)}{c'(\kappa)\kappa_m}$. Thus, the sufficient condition for every firm's profit to decline as a result of trade is $-m\kappa'(m) > \frac{c(\kappa)}{c'(\kappa)}$.

In endogenous labor, for firm j , the demand is $p(q(j)) = f'(Q)u'(q(j))$, then its profit is $\pi(j) = mq(j)p(j) - mq(j)c(j) = mq(j)f'(Q)u'(q(j)) - mq(j)c(j)$. Differentiate with respect to m and use Envelope Theorem, $\pi_m(j) = q(j)u'(q(j))[f'(Q) + mf''(Q)Q_m] - c(j)q(j)$. Therefore, a sufficient condition for firm j 's profit to decline is $f'(Q) + mf''(Q)Q_m < 0$. Since $f'(Q)u'(0) = c(\kappa)$, hence $u'(0)f''(Q)Q_m = c'(\kappa)\kappa_m$, so $f'(Q) + mf''(Q)Q_m = \frac{1}{u'(0)}[c(\kappa) + mc'(\kappa)\kappa_m]$; or $-m\kappa'(m) > \frac{c(\kappa)}{c'(\kappa)}$. Q.E.D.

Proof of Corollary 1

To show that no-entry is the unique equilibrium when, conditional on $\alpha = 0$, trade reduces the profitability of the tradable sector, note that the average profit is

$$\bar{\pi}(\alpha) = \int_0^{\kappa(\alpha)} \left[m \frac{1}{\lambda(\alpha)} u'(q(j)) q(j) - \int_0^{mq(j)} c(q, j) dq \right] \phi(j) dj,$$

when labor is in fixed supply. Differentiating $\bar{\pi}(\alpha)$ with respect to α , and making use of the facts that the marginal firm makes zero profit and that λ is increasing in α ,²¹ we have

$$\frac{d\bar{\pi}(\alpha)}{d\alpha} = \int_0^{\kappa(\alpha)} m \frac{d\frac{1}{\lambda(\alpha)}}{d\alpha} u'(q(j)) q(j) \phi(j) dj < 0, \forall \alpha.$$

We can apply the same exercise for the case of endogenous labor and draw the same conclusion that $\frac{d\bar{\pi}(\alpha)}{d\alpha} < 0, \forall \alpha$. Q.E.D.

²¹Since κ and $q(j), j \in [0, \kappa]$ are decreasing in λ , and by condition (7), κ and $q(j), j \in [0, \kappa]$ must be decreasing in α . Hence λ is increasing in α .

Proof of Proposition 6

If marginal cost is constant, then $c(Nq(j), j) = c(j)$. None of the three equilibrium equations depends on N , therefore the equilibrium κ is invariant to N .

Now suppose that marginal cost is increasing in output. We prove by contradiction that $Q^N < Q^c$. Suppose that $Q^N \geq Q^c$, then $f'(Q^N) \leq f'(Q^c)$. By equation (11), we have $\kappa^N \leq \kappa^c$. Meanwhile, $f'(Q^N) \leq f'(Q^c)$ implies that $q^N(j) < q^c(j)$ for all $j \leq \kappa^N$. This is because, otherwise $Nq^N(j) > q^c(j)$ and hence $c(Nq^N(j), j) > c(q^c(j), j)$, making it impossible for equation (10) to hold both before and after the expansion of the population size. Since $\kappa^N \leq \kappa^c$ and $q^N(j) < q^c(j)$ for all $j \leq \kappa^N$, we have $Q^N = \int_0^{\kappa^N} u(q^N(j))dj < \int_0^{\kappa^c} u(q^c(j))dj = Q^c$, contradicting the assumption that $Q^N \geq Q^c$. Finally note that $Q^N < Q^c$ implies $\kappa^N > \kappa^c$. Q.E.D.

Proof of Proposition 7

When the marginal cost is constant in output, none of the three equilibrium equations (13) through (15) depends on N , therefore the equilibrium κ is invariant to N .

When the marginal cost is increasing in output, we note that, according to equation (14), less efficient firms are crowded out (in) if and only if the shadow price of labor λ increases (decreases). Thus, we fix λ and differentiate the left hand side of equation (15) with respect to N . If the derivative is larger than l , the increase in demand for labor as a result of the expansion in consumer base must outpace the increase in supply of the labor force, thus leading to an increase in λ . Otherwise, a drop in λ .

The derivative of the left hand side of equation (15) is

$$\int_0^{\kappa} c(Nq(j), j) \left[q(j) + N \frac{dq(j)}{dN} \right] dj. \quad (21)$$

By equation (13), we have

$$r' \frac{dq(j)}{dN} = \lambda c'(Nq(j)) \left[q(j) + N \frac{dq(j)}{dN} \right],$$

and hence

$$\frac{dq(j)}{dN} = \frac{\lambda c'(Nq(j))q(j)}{r'(q(j)) - \lambda c'(Nq(j))N},$$

which in turn implies that

$$q(j) + N \frac{dq(j)}{dN} = \frac{q(j)}{1 - \lambda N \frac{c'(Nq(j))}{r'(q(j))}}.$$

Substituting the above into equation (21), we are able to rewrite the derivative of the left hand side

of equation (15) as

$$\int_0^{\kappa} c(Nq(j), j)q(j) \frac{1}{1 - \lambda N \frac{c'(Nq(j))}{r'(q(j))}} dj.$$

Substituting equation (13) into the above, we obtain the derivative of the left hand side of equation (15) as

$$\int_0^{\kappa} c(Nq(j), j)q(j) \frac{1}{1 - n \frac{r(q(j))}{c(Nq(j))} \frac{c'(Nq(j))}{r'(q(j))}} dj.$$

Evaluating this at $N = 1$, we have

$$\int_0^{\kappa} c(q^c(j), j)q^c(j) \frac{1}{1 - \frac{r(q^c(j))c'(q^c(j))}{c(q^c(j))r'(q^c(j))}} dj.$$

Since $\int_0^{\kappa} c^c(q(j), j)q^c(j)dj$ is finite and is bounded below by l , we can then conclude that the derivative of the left hand side of equation (15) is greater than l if $-\frac{r(q^c(j))c'(q^c(j))}{c(q^c(j))r'(q^c(j))}$ is sufficiently small for all j , and is less than l if $-\frac{r(q^c(j))c'(q^c(j))}{c(q^c(j))r'(q^c(j))}$ is sufficiently large for all j . The former case in turn entails an increase in λ and hence $\kappa^N < \kappa^c$, and the latter case vice versa. Q.E.D.

Proof of Proposition 9

Consider first the case of endogenous labor. Suppose that $w^x f'(Q^x) = w^y f'(Q^y)$ for any destination countries x, y .

Recall that consumers in destination country x consume $q_h^x(j)$ from firm j of origin country h such that

$$w^x f'(Q^x)r(q_h^x(j)) = w^h c_h \left(\sum_{z=1}^m q_h^z(j), j \right), \text{ for all } j \leq \kappa_h^x$$

where κ_h^x is the last variety bought by x from h and is determined by

$$w^x f'(Q^x)u'(0) = w^h c_h \left(\sum_{z=1}^m q_h^z(\kappa_h^x), \kappa_h^x \right),$$

and $\sum_{z=1}^m q_h^z(\kappa_h^x)$ is the global output of firm κ_h^x from origin country h . Notice that although κ_h^x is the last variety sold by h to x , it is not necessarily the last variety sold by h globally. Hence it is possible that $\sum_{z=1}^m q_h^z(\kappa_h^x) > 0$.

However, given that $w^x f'(Q^x)$ is identical for all destination countries, we have $q_h^x(j)$ and κ_h^x must be the same for all destination country x (and hence $\sum_{z=1}^m q_h^z(\kappa_h^x) = 0$). This in turn implies that Q^x must be the same for all destination countries, and hence $w^x = w^y$ for all x, y . As a result,

every destination country must spend the same amount of expenditure:

$$\sum_{h=1}^m \int_0^{\kappa_h^x} w^x f'(Q^x) u'(q_h^x(j)) q_h^x(j) dj = \sum_{h=1}^m \int_0^{\kappa_h^y} w^y f'(Q^y) u'(q_h^y(j)) q_h^y(j) dj, \quad \forall x, y.$$

Meanwhile, given that wage rate is the same across all origin countries, the above equilibrium condition becomes

$$f'(Q)r(q_h(j)) = c_h(mq_h(j), j), \quad \text{for all } j \leq \kappa_h.$$

Since all origin countries face the same set of demand, we have $q_x(j) = q_y(j)$ if and only if $c_x(q, j) = c_y(q, j)$. Accordingly, the origin country that is more productive must produce more and hence (given positive marginal revenue) earn more revenue than the origin country that is less productive. As a result, trade cannot be balanced. Contradiction.

Now turn to the fixed labor case. We begin with the following lemma.

Lemma A. 1 *Suppose that country x dominates country y in productivity. Then*

$$\sum_{h=1}^m q_x^h(j) \geq \sum_{h=1}^m q_y^h(j)$$

for all j , with strict inequalities for $\sum_{h=1}^m q_x^h(j) > 0$.

Proof. Suppose otherwise is true. Then, there must exist j^* such that

$$\sum_{h=1}^m q_x^h(j^*) < \sum_{h=1}^m q_y^h(j^*),$$

and hence

$$c_x\left(\sum_{h=1}^m q_x^h(j^*), j^*\right) < c_y\left(\sum_{h=1}^m q_y^h(j^*), j^*\right).$$

Since the following equilibrium conditions hold,

$$\begin{aligned} \frac{w^h}{\lambda^h w^x} r(q_x^h(j^*)) &= c_x\left(\sum_{h=1}^m q_x^h(j^*), j^*\right), \\ c_y\left(\sum_{h=1}^m q_y^h(j^*), j^*\right) &= \frac{w^h}{\lambda^h w^y} r(q_y^h(j^*)); \end{aligned}$$

we conclude that $w^x > w^y$ (given $\sum_{h=1}^m q_x^h(j^*) < \sum_{h=1}^m q_y^h(j^*)$, it is not possible that $r(q_x^h(j^*)) < r(q_y^h(j^*))$ while $w^x \leq w^y$ as it would imply that $q_x^h(j^*) \geq q_y^h(j^*)$ for all h , contradicting the assumption that $\sum_{h=1}^m q_x^h(j^*) < \sum_{h=1}^m q_y^h(j^*)$).

Meanwhile, in order for the labor market to clear in both countries of the same size, there must also exist j' such that

$$\sum_{h=1}^m q_x^h(j') > \sum_{h=1}^m q_y^h(j'),$$

such that

$$c_x(Q_x, j') > c_y(Q_y, j').$$

Since the above equilibrium conditions hold for j' as well, we conclude that $w^x < w^y$ (otherwise, we have $r(q_x^h(j^*)) > r(q_y^h(j^*))$ and hence $q_x^h(j^*) \leq q_y^h(j^*)$ for all h , contradicting the assumption that $\sum_{h=1}^m q_x^h(j^*) > \sum_{h=1}^m q_y^h(j^*)$). Contradiction. \blacksquare

Having established the lemma, given that $\sum_{h=1}^m q_x^h(j) \geq \sum_{h=1}^m q_y^h(j)$ when x dominates y in productivity, we have by revealed preferences that firms in country x must make (weakly) more profits than corresponding firms in country y , and strictly so for some j , and hence country x must earn more revenue than country y . Meanwhile, given that $w^x f'(Q^x) = w^y f'(Q^y)$ for all x and y , all destination countries must have the same spending. Hence, trade cannot be balanced. Contradiction. *Q.E.D.*

Proof of Proposition 10

Consider first the fixed labor case. We prove by contradiction. Suppose that x dominates y in productivity (and hence $c_x(q, j) < c_y(q, j)$ for any given q and j) such that $\frac{w^x}{\lambda^x} \leq \frac{w^y}{\lambda^y}$. Since the two countries face the same set of firms in the world, $\frac{w^x}{\lambda^x} \leq \frac{w^y}{\lambda^y}$ implies that $q_h^x(j) \leq q_h^y(j)$ for all j in all h . Since the marginal revenue is positive for all equilibrium non-zero choices of output and since $\frac{w^x}{\lambda^x} \leq \frac{w^y}{\lambda^y}$, we can conclude that the revenue generated from $q_h^x(j)$ must be weakly smaller than that from $q_h^y(j)$ for all j in all h . This in turn implies that the consumption spending of country x must not exceed that of country y . Meanwhile, because the marginal revenue is positive for all non-zero output, and since both countries face the same set of demand, Lemma A.1 implies that the total revenue received by firms in country x must exceed that in country y . Contradiction.

Consider next the endogenous labor case. Once again, suppose that there exists two countries, $x < y$ such that $w^x f'(Q^x) \leq w^y f'(Q^y)$. By the same logic as before, $q_h^x(j) \leq q_h^y(j)$ for all j in all h , and the consumption spending of country x must not exceed that of country y . Moreover, because $q_h^x(j) \leq q_h^y(j)$ for all j in all h , we have $Q^x \leq Q^y$, which implies that $w^x \leq w^y$. This, together with the fact that country x dominates country y in productivity, suggests that $q_x^h(j) \geq q_y^h(j)$ for all j in all h . Since the marginal revenue is positive for all non-zero output, and since both countries face

the same set of demand, this in turn implies that the total revenue received by firms in country x must exceed that in country y . Hence, trade cannot be balanced. Q.E.D.

Proof of Proposition 12

The first two points are obvious and hence their proofs are omitted.

The third observation is based on the fact that countries dominated by country h in productivity will have smaller aggregator and higher production cost. Therefore, if country y specializes in export, and hence its marginal revenue from serving the domestic market is smaller than the marginal cost of doing so, all countries that are dominated by country y must not serve their domestic markets either.

The fourth observation is obtained because countries within the rankable subset that do not dominate country y in productivity must have a (weakly) smaller aggregator. Thus, if firms in country y do not serve their own domestic market, they must not serve those foreign markets either, hence no trade.

To see the last point, suppose that $w^x \leq w^y$. Since $w^x f'(Q^x) > w^y f'(Q^y)$ (Proposition 10), this implies that $Q^x < Q^y$. Meanwhile, since x and y face the same set of supplying firms worldwide and since $w^x f'(Q^x) > w^y f'(Q^y)$, we have $q_h^x(j) \geq q_h^y(j)$ for all h and j , with strict inequalities for some j and hence $Q^x > Q^y$. Contradiction. Q.E.D.

Proof of Proposition 13

Since all countries are assumed to be symmetric in productivity but differ in size, equations (16) and (18) can be rewritten as

$$\frac{w^i}{\lambda^i} r(q_h^i(j)) = w^h c\left(\sum_{z=1}^m N^z q_h^z(j), j\right), \quad (22)$$

for all $q_h^i(j) > 0$.

We first prove the statements for the fixed labor case. To show that $w^x < w^y$ when $N^x > N^y$, suppose otherwise is true. Then, given that both countries face the same demand in all destination countries (including themselves), $w^x \geq w^y$ implies that

$$\sum_{z=1}^m N^z q_x^z(j) \leq \sum_{z=1}^m N^z q_y^z(j)$$

for all j . In other words, the total output of each firm in the larger country must be (weakly) smaller than that of the smaller country. This is because otherwise, there exists \tilde{j} such that

$$\sum_{z=1}^m N^z q_x^z(\tilde{j}) > \sum_{z=1}^m N^z q_y^z(\tilde{j})$$

for all $q_h^i(\tilde{j}) > 0$. According to equation (22), this in turn implies that $q_x^i(\tilde{j}) < q_y^i(\tilde{j})$ for all i such that $q_x^i(\tilde{j}) > 0$, which further implies that $\sum_{z=1}^m N^z q_x^z(\tilde{j}) < \sum_{z=1}^m N^z q_y^z(\tilde{j})$, a contradiction.

Meanwhile, labor market clearing in each of the origin countries requires

$$N^i l = \int_0^\infty \int_0^{\sum_{z=1}^m N^z q_i^z(j)} c(q, j) dq dj.$$

Since $N^x > N^y$ and $\sum_{z=1}^m N^z q_x^z(j) \leq \sum_{z=1}^m N^z q_y^z(j)$ for all j , the labor market cannot be cleared in both countries. Contradiction.

To show that $\frac{w^x}{\lambda^x} < \frac{w^y}{\lambda^y}$ when $N^x > N^y$, suppose otherwise is true. Then, since these two destination countries face the same set of supplies across the world, we have as per equation (22) that each consumer in country x must consume (weakly) more than his counterpart in country y : $q_h^x(j) \geq q_h^y(j)$ for all h and all j . This in turn implies that each consumer in country x must spend more. Since $N^x > N^y$, we have total spending of country x to be more than $\frac{N^x}{N^y}$ times that of country y .

Given that trade must be balanced, the total spending of a country must equal its total income. This in turn implies that the total revenue made by all firms in country x must be more than $\frac{N^x}{N^y}$ times that of all firms in country y . Meanwhile, since

$$N^x = \int_0^\infty \int_0^{\sum_{z=1}^m N^z q_x^z(j)} c(q, j) dq dj, \quad N^y = \int_0^\infty \int_0^{\sum_{z=1}^m N^z q_y^z(j)} c(q, j) dq dj,$$

the total cost of producing the income that is more than $\frac{N^x}{N^y}$ times that of country y is exactly $\frac{N^x}{N^y}$ times the total cost incurred in country y . Since the two countries face the same set of downward sloping marginal revenues and the same (weakly) diminishing returns to scale technology, this is impossible.

Turning now to the endogenous labor case, we once again show by contradiction that $w^x < w^y$ when $N^x > N^y$. Suppose otherwise is true. Then, given that both origin countries face the same demand in all destination countries (including themselves) and that both origin countries have the same technologies, $w^x \geq w^y$ implies that each firm in the larger origin country x must make (weakly) smaller total revenue, and strictly so for some of them. Accordingly, the larger origin

country x must have a smaller income, which means that the larger destination country x must have a smaller spending. Given that $N^x > N^y$, each consumer's spending must be much smaller in the larger destination country x than that in the smaller destination country y . Since all consumers face the same set of firms around the globe, a consumer's spending will be smaller in destination country x than in destination country y if and only if $w^x f'(Q^x) < w^y f'(Q^y)$. This in turn has two implications. First, $q_h^x(j) \leq q_h^y(j)$ for all h and j as per equation (22), with strict inequality for at least some q and j . Hence, $Q^x < Q^y$. Second, since $w^x \geq w^y$, this also implies that $f'(Q^x) < f'(Q^y)$ and hence $Q^x > Q^y$, contradiction.

Given that $w^x < w^y$ when $N^x > N^y$, we can show further that $w^x f'(Q^x) < w^y f'(Q^y)$. Suppose otherwise is true: $w^x f'(Q^x) \geq w^y f'(Q^y)$. This has two implications: First, since consumers in these two destination countries face the same set of firms, $w^x f'(Q^x) \geq w^y f'(Q^y)$ implies that $q_h^x(j) \geq q_h^y(j)$ for all h and j per equation (22), with strict inequality for at least some q and j . Hence $Q^x > Q^y$. Second, since $w^x < w^y$, $w^x f'(Q^x) \geq w^y f'(Q^y)$ also implies that $Q^x < Q^y$. Contradiction. Q.E.D.

Proof of Lemma 1

Differentiating $d_h(i)$ with respect to i to establish that $d_h(i)$ increase in i , i.e., more productive firms (smaller i) will export more, if and only if

$$\frac{\partial q_h^h(i)}{\partial i} \sum_{x \neq h} q_h^x(i) > q_h^h(i) \sum_{x \neq h} \frac{\partial q_h^x(i)}{\partial i}. \quad (23)$$

Without loss of generality, consider the fixed labor case. Differentiating (16) with respect to i , we have

$$\begin{aligned} \frac{w^x}{\lambda^x} \frac{\partial r}{\partial q_y^x} \frac{\partial q_y^x}{\partial i} &= w^y \frac{\partial c_y}{\partial q} \frac{\partial q_y^h}{\partial i} + w^y \frac{\partial c_y}{\partial i} \\ &= w^y \frac{\partial c_y}{\partial i} \quad \text{given constant marginal cost.} \end{aligned}$$

Making use of the first order condition: $\frac{w^x}{\lambda^x} r(q_y^x) = w^y c_y$, we have $\frac{\partial q_y^x}{\partial i} = \frac{\frac{\partial c_y}{\partial i} r(q_y^x)}{c_y \frac{\partial r}{\partial q_y^x}}$. Substituting this into (23),

$$\frac{\frac{\partial c_h}{\partial i} r(q_h^h)}{c_h \frac{\partial r}{\partial q_h^h}} \sum_{x \neq h} q_h^x(i) > q_h^h(i) \sum_{x \neq h} \frac{\frac{\partial c_h}{\partial i} r(q_h^x)}{c_h \frac{\partial r}{\partial q_h^x}}.$$

After simplification, we can rewrite the condition as:

$$\sum_{x \neq h} q_h^x(i) q_h^h(i) \left[\frac{\frac{r(q_h^h(i))}{q_h^h(i)}}{\frac{\partial r}{\partial q_h^h(i)}} - \frac{\frac{r(q_h^x(i))}{q_h^x(i)}}{\frac{\partial r}{\partial q_h^x(i)}} \right] > 0;$$

or

$$\sum_x q_h^x(i) q_h^h(i) \left[\frac{1}{\frac{\partial \ln r}{\partial \ln q_h^h(i)}} - \frac{1}{\frac{\partial \ln r}{\partial \ln q_h^x(i)}} \right] > 0.$$

Since $\frac{\partial r}{\partial q} < 0$, the above inequality can be rewritten as:

$$\sum_x q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right] > 0.$$

Q.E.D.

Proof of Proposition 15

Following equations (16) and (18), we can conclude that a country with a larger aggregator will give any firm from the globe a larger sale in this country as destination market. Accordingly, for any country h , there exists a sequence of thresholds $\{\kappa_h^x\}_{x=1,2,\dots,m}$ with $\kappa_h^x < \kappa_h^{x+1}$ for all x , such that $q_h^x(i) > 0$ if and only if $i < \kappa_h^x$.

Country m has the largest aggregator. Therefore, $q_m^m(i) > q_m^x(i)$ for all x and for all i such that $q_m^m(i) > 0$. Given the assumption that $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q , this implies that

$$\frac{1}{\left| \frac{\partial \ln r(q_m^x(i))}{\partial \ln q_m^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_m^m(i))}{\partial \ln q_m^m(i)} \right|} > 0,$$

for all x and for all i such that $q_m^m(i) > 0$ and $q_m^x(i) > 0$. This in turn implies that, for $i < \kappa_m^{m-1}$ and hence $q_m^x(i) > 0$,

$$\sum_x q_m^x(i) q_m^m(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_m^x(i))}{\partial \ln q_m^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_m^m(i))}{\partial \ln q_m^m(i)} \right|} \right] > 0.$$

Therefore, $e_m(i)$ decreases in i for $i \leq \kappa_m^{m-1}$. For $i > \kappa_m^{m-1}$, $q_m^x(i) = 0$ for all $x \neq m$. Therefore, $e_m(i) = 0$ for $i \in (\kappa_m^{m-1}, \kappa_m^m]$.

For country h with a smaller aggregator, $q_h^h(i) = 0$ for all $i > \kappa_h^h$, whereas firm $i > \kappa_h^{h-1}$ does not

export to any country that has a smaller aggregator than country h , and hence $q_h^x(i) = 0$. Therefore, for $i \in [\kappa_h^{h-1}, \kappa_h^h]$,

$$\begin{aligned} & \sum_x q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right] \\ &= \sum_{x=h}^m q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right] \\ &= \sum_{x=h+1}^m q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right]. \end{aligned}$$

As $q_h^x(i) > q_h^h(i)$ for all $x > h$ and for all i such that $q_h^h(i) > 0$, given the assumption that $\left| \frac{\partial \ln r(q)}{\partial \ln q} \right|$ increases in q , we have:

$$\sum_{x=h+1}^m q_h^x(i) q_h^h(i) \left[\frac{1}{\left| \frac{\partial \ln r(q_h^x(i))}{\partial \ln q_h^x(i)} \right|} - \frac{1}{\left| \frac{\partial \ln r(q_h^h(i))}{\partial \ln q_h^h(i)} \right|} \right] < 0.$$

Therefore, $e_h(i)$ strictly increases in i for $i \in [\kappa_h^{h-1}, \kappa_h^h]$. Since $e_h(i)$ is continuously differentiable in i , there must exist $\kappa(h) \in [0, \kappa_h^{h-1}]$ such that $e_h(i)$ increases in i for $i \in [\kappa(h), \kappa_h^h]$.

For all $i \in (\kappa_h^h, \kappa_h^m]$, $q_h^h(i) = 0$ whereas $q_h^x(i) = 0$ for some $x \in (h, m]$. Therefore, $e_h(i) = 1$.

Q.E.D.

Proof of Corollary 3

Recall that $r(q) = u'(q) + qu''(q)$. Hence

$$\left| \frac{\partial \ln r(q)}{\partial \ln q} \right| = \frac{-\frac{\partial r}{\partial q}}{\frac{r(q)}{q}} = -\frac{-2u''(q) + qu'''(q)}{\frac{u'(q) + qu''(q)}{q}}.$$

Dividing both the numeraire and the denominator by $-u''$, we have

$$\left| \frac{\partial \ln r(q)}{\partial \ln q} \right| = \frac{2 + q \frac{u'''(q)}{u''(q)}}{-\frac{u'(q)}{u''(q)q} - 1}.$$

Suppose that u is CARA. Then

$$\frac{u''(q)}{u'(q)} = -c,$$

implying that

$$\frac{u'''(q)u'(q) - u''(q)u''(q)}{u'^2} = 0,$$

or

$$\frac{u'''(q)}{u''(q)} = \frac{u''(q)}{u'(q)} = -c.$$

Hence

$$\left| \frac{\partial \ln r(q)}{\partial \ln q} \right| = \frac{2 - qc}{\frac{1}{cq} - 1} = \frac{(2 - qc)qc}{1 - cq}.$$

Our assumption that $\frac{\partial r}{\partial q} < 0$ implies that $2 - qc > 0$, whereas $r(q) > 0$ in equilibrium implies that $1 - qc > 0$. Using these properties, we can show that $\frac{(2 - qc)qc}{1 - cq}$ is increasing in q . Q.E.D.

Proof of Lemma 2

Suppose not. By the following equilibrium conditions,

$$\frac{1}{\lambda^x} u'(0) = c_x \left(\sum_{h \neq x}^m q_x^h(j), \kappa_x^x \right); \quad \frac{1}{\lambda_x^c} u'(0) = c_x(0, \kappa_x^c) \quad \text{if fixed supply of labor;}$$

$$f'(Q^x) u'(0) = c_x \left(\sum_{h \neq x}^m q_x^h(j), \kappa_x^x \right); \quad f'(Q_x^c) u'(0) = c_x(0, \kappa_x^c) \quad \text{if endogenous supply of labor;}$$

we have $\frac{1}{\lambda^x} \geq \frac{1}{\lambda_x^c}$ in the case of fixed labor and $f'(Q^x) \geq f'(Q_x^c)$ in the case of endogenous labor.

Consider the case of fixed labor, since

$$\begin{aligned} \frac{1}{\lambda^x} r(q_x^x(j)) &= c_x \left(\sum_{h=1}^m q_x^h(j), j \right) \\ \frac{1}{\lambda_x^c} r(q_x^c(j)) &= c_x(q_x^c(j), j) \end{aligned}$$

for all $j \leq \kappa_x^c$. Then $\frac{1}{\lambda^x} \geq \frac{1}{\lambda_x^c}$ implies that $\sum_{h=1}^m q_x^h(j) \geq q_x^c(j)$ for all $j \leq \kappa_x^c$, suggesting that labor market cannot be cleared both before and after trade.

Consider the case of endogenous labor, $f'(Q^x) \geq f'(Q_x^c)$ implies $Q^x \leq Q_x^c$. Since the following

equilibrium conditions hold

$$\begin{aligned} f'(Q^x)r(q_x^x(j)) &= c_x\left(\sum_{z=1}^m q_x^z(j), j\right) \\ f'(Q_x^c)r(q_x^c(j)) &= c_x(q_x^c(j), j) \end{aligned}$$

we have, for any $j \leq \kappa_x^c$, either (1) $q_x^x(j) > q_x^c(j)$ or (2) $q_x^x(j) \leq q_x^c(j)$ but $\sum_{h=1}^m q_x^h(j) \geq q_x^c(j)$.

Define J_1 as the set of j such that $q_x^x(j) > q_x^c(j)$ and J_2 as the set of j such that $q_x^x(j) \leq q_x^c(j)$ but $\sum_{h=1}^m q_x^h(j) \geq q_x^c(j)$.

Lemma A. 2 Fixing $f'(Q^x) > f'(Q_x^c)$, $\sum_{h=1}^m u(q_h^x(j)) > u(q_x^c(j))$ for any $j \in J_2$ in a trade equilibrium.

Proof. Consider any given $j \in J_2$. Let $\Delta(j) = \sum_{h=1}^m q_x^h(j) - q_x^x(j)$ be the amount of output produced by firm j of country x that is not sold domestically. Firm j of country x earns two parts of revenue, one domestically and one from abroad. The amount of revenue earned abroad must be at least as large as $f'(Q^x)r(q_x^x)\Delta(j)$. This is because the marginal revenue earned abroad must be equal to the marginal cost $c_x(\sum_{h=1}^m q_x^h(j), j)$; since $c_x(\sum_{h=1}^m q_x^h(j), j) = f'(Q^x)r(q_x^x)$, the marginal revenue earned abroad must be equal to $f'(Q^x)r(q_x^x)$. Accordingly, the total revenue earned by firm j of country x is at least

$$f'(Q^x)\left(\int_0^{q_x^x} r(q)dq + r(q_x^x)\Delta(j)\right).$$

Fixing consumers' labor spending on j : $\int_0^{\sum_{h=1}^m q_x^h(j)} c_x(q, j)dq$, the income thus earned by consumers in country x , denoted by $I(j)$, must be equal to the revenue generated by firm j of country x (recall that firms' profits accrue to domestic consumers) and hence

$$I(j) \geq f'(Q^x)\left(\int_0^{q_x^x} r(q)dq + r(q_x^x)\Delta(j)\right).$$

Consider the consumption choice of a representative consumer (with the population size normalized to unity) given the income constraint $I(j)$ and the rest of her consumption bundle $q_h^x(i)$ for all h and $i \neq j$:

$$\max \sum_{h=1}^m u(q_h^x(j)),$$

subject to

$$I(j) = \sum_{h=1}^m p_h(j)q_h^x(j),$$

where $p_h(j)$ is the equilibrium price of variety j from country h . From the first order condition, we

have, for all h, i ,

$$\frac{u'(q_h^x(j))}{u'(q_i^x(j))} = \frac{p_h(j)}{p_i(j)}.$$

Define $\Delta(j)_h$ such that $q_h^x(j) = \frac{p_x(j)}{p_h(j)}\Delta(j)_h$ to be the amount of $q_x^h(j)$ equivalent when firm j of country x sells $q_x^h(j)$ of its product in country h . Accordingly,

$$\begin{aligned} \max \sum_{h=1}^m u(q_h^x(j)) &\geq u(q_x^x(j)) + \sum_{h \neq x}^m q_h^x u'(q_h^x(j)) \\ &= u(q_x^x(j)) + \sum_{h \neq x}^m \Delta(j)_h \frac{p_x(j)}{p_h(j)} u'(q_h^x(j)) \\ &= u(q_x^x(j)) + \sum_{h \neq x}^m \Delta(j)_h \frac{u'(q_x^x(j))}{u'(q_h^x(j))} u'(q_h^x(j)) \\ &= u(q_x^x(j)) + u'(q_x^x(j)) \sum_{h \neq x}^m \Delta(j)_h. \end{aligned}$$

In equilibrium, trade must be balanced. Since all the rest of choices are in equilibrium, trade balance requires $\sum_{h \neq x}^m \Delta(j)_h = \Delta(j)$. Hence, we have

$$\max \sum_{h=1}^m u(q_h^x(j)) \geq u(q_x^x(j)) + u'(q_x^x(j))\Delta(j).$$

Recall that $j \in J_2$, hence $u'(q_x^c(j)) > u'(q_x^x(j))$ and $\Delta(j) > q_x^c(j) - q_x^x(j)$. We therefore conclude that

$$\max \sum_{h=1}^m u(q_h^x(j)) \geq u(q_x^c(j)).$$

■

Having established the lemma, since

$$\begin{aligned} Q^x &= \int_0^\infty \sum_{h=1}^m u(q_h^x(j))dj = \int_{J_1} \sum_{h=1}^m u(q_h^x(j))dj + \int_{J_2} \sum_{h=1}^m u(q_h^x(j))dj \\ &> \int_{J_1} u(q_x^c(j))dj + \int_{J_2} u(q_x^c(j))dj \\ &= Q_x^c, \end{aligned}$$

we have $f'(Q^x) < f'(Q_x^c)$. Contradiction.

Q.E.D.

Proof of Proposition 16

Recall that the following equilibrium conditions hold under autarky for country y :

$$\frac{1}{\lambda_y^c} r(q_y^c(j)) = c_y(N^y q_y^c(j), j); \quad (24)$$

$$\frac{1}{\lambda_y^c} u'(0) = c_y(0, \kappa_y^c); \quad (25)$$

$$N^y l = \int_0^\infty \int_0^{N^y q_y^c(j)} c_y(q, j) dq dj \quad (26)$$

In comparison, the following equilibrium conditions hold under trade for the same country,

$$\frac{w^h}{\lambda^h} r(q_y^h(j)) = w^y c_y\left(\sum_{i=1}^m N^i q_y^i(j), j\right), \quad (27)$$

$$\frac{w^h}{\lambda^h} u'(0) = w^y c_y(0, \kappa_y^h), \quad (28)$$

$$N^y l = \int_0^\infty \int_0^{\sum_{i=1}^m N^i q_y^i(j)} c_y(q, j) dq dj, \quad (29)$$

for all $h = 1, 2, \dots, m$.

Suppose that there exists z such that $\kappa_y^z \geq \kappa_y^c$ (that is, trade does not crowd out inefficient firms in country y due to the presence of country z), by conditions (25) and (28) we have $\frac{w^h}{w^y \lambda^z} > \frac{1}{\lambda_y^c}$. This in turn implies that, according to conditions (24) and (27), either for all $j < \kappa_y^z$,

$$\sum_{i=1}^m N^i q_y^i(j) \geq N^y q_y^c(j);$$

or when there exists some $j < \kappa_y^z$ such that $\sum_{i=1}^m N^i q_y^i(j) \leq N^y q_y^c(j)$, and

$$q_y^z(j) \geq q_y^c(j).$$

The former case is not possible because either the labor market cannot be cleared both under trade and under autarky (if $\sum_{i=1}^m N^i q_y^i(j) > N^y q_y^c(j)$ for all $j < \kappa_y^z$), or otherwise (if $\sum_{i=1}^m N^i q_y^i(j) = N^y q_y^c(j)$ for some $j < \kappa_y^z$) it implies that $q_y^h(j) = q_y^c(j)$ for all $h = 1, 2, \dots, m$, which contradicts the assumption that $\sum_{i=1}^m N^i q_y^i(j) = N^y q_y^c(j)$ for some $j < \kappa_y^z$.

The latter case is not possible if all countries are of the same size as $q_y^z(j) \geq q_y^c(j)$ implies $N^y q_y^h(j) \geq N^y q_y^c(j)$, contradicting $\sum_{i=1}^m N^i q_y^i(j) \leq N^y q_y^c(j)$. Therefore, we conclude that asymmetric trade crowds out less productive firms if all countries are of the same size. Q.E.D.

Proof of Proposition 17

The tradable sector in country y will be missing under autarky if and only if

$$f'(0)u'(0) < \beta c(0,0).$$

Define β_1 such that

$$\beta_1 \equiv \frac{f'(0)u'(0)}{c(0,0)}.$$

Given the assumption that $f'(0)u'(0) > c(0,0)$, the more productive country x has an active tradable sector for all β . If the tradable sector of country y remains inactive in a trade equilibrium, given the equilibrium wage rate elsewhere, including that of country x that dominates y in productivity, the wage rate of country y must ensure that country y and country x do not trade with each other. That is,

$$w^x f'(Q^x)u'(0) < w^y \beta c(0,0); \text{ and } w^y f'(0)u'(0) < w^x c(q_x,0).$$

In the above condition, Q^x is the consumption composite attained by country x in the supposed trade equilibrium, and q_x is the total output produced by the most productive firm in country x in that equilibrium. That is,

$$\frac{w^x c(q_x,0)}{f'(0)u'(0)} < w^y < \frac{w^x f'(Q^x)u'(0)}{\beta c(0,0)}.$$

Note that, in the supposed trade equilibrium, Q^x and q_x are both independent of β since the tradable sector of country y does not operate at all.

We can then conclude that there exists a trading equilibrium where the tradable sector of the less productive country y remains missing as in autarky only if,

$$\beta > \max \left\{ \beta_1, \frac{f'(Q^x)u'(0)}{c(q_x,0)} \frac{f'(0)u'(0)}{c(0,0)} \right\}.$$

Define β_2 such that

$$\beta_2 \equiv \frac{f'(Q^x)u'(0)}{c(q_x,0)} \frac{f'(0)u'(0)}{c(0,0)}.$$

Since the more productive country x has an active tradable sector for all β , we have:

$$f'(Q^x)u'(0) > c(q_x,0).$$

Hence

$$\beta_2 \equiv \frac{f'(Q^x)u'(0)}{c(q_x, 0)} \frac{f'(0)u'(0)}{c(0, 0)} > \beta_1.$$

Now consider $\beta \in (\beta_1, \beta_2]$. Evidently, the tradable sector will be missing in country y under autarky, but it cannot be missing in a trade equilibrium. Furthermore, note that when $\beta = \beta_1$, the tradable sector of country y has a positive measure of operating firms in trade but is missing under autarky. Therefore, by continuity, we can conclude that there must exist $\beta_0 < \beta_1$ such that for all $\beta \in [\beta_0, \beta_2]$, trade crowds in less productive firms in country y : $\max_{h \neq y} \{\kappa_y^h\} > \kappa_y^c$.

Finally, since all those crowded in must be specialized in export as per Corollary 4, the following condition holds in the trade equilibrium attained at β_0 :

$$\max_{h \neq y} \left\{ \frac{w^h}{w^y} f'(Q^h(\beta_0))u'(0) \right\} = c(0, \max_{h \neq y} \{\kappa_y^h\}) > f'(Q^y(\beta_0))u'(0).$$

Note that $Q^y(\beta_2)$ (and $Q^h(\beta_2)$) is the consumption composite of country y (and country h respectively) in such an equilibrium. The condition in turn implies that there must exist firms in country y that are more productive than firm $\max_{h \neq y} \{\kappa_y^h\}$ but are also specialized in trade. By continuity, we can then conclude that there exists $\beta_{-1} < \beta_0$ such that, for $\beta \in [\beta_{-1}, \beta_2]$, at least some less productive firms in the tradable sector of country y are specialized in export. Q.E.D.

Proof of Proposition 18

We prove by constructing a preference that ensures crowding-in and at the same time satisfies Properties (3*) and (4*). Let the number of x countries be m_x and that of y countries be m_y . To simplify the notation, we drop the subscript x from $c_x(\cdot)$, the marginal cost function of an x country, and normalize the wage rate of y country in a trade equilibrium to be one.

First, note that regardless of trade or autarky, if κ is the marginal firm of a particular country serving a market, then in that market, the output of firm j , $q(j)$, will be given by

$$\frac{r(q(j))}{u'(0)} = \frac{c(q(j), j)}{c(0, \kappa)}.$$

That is, conditional on a threshold κ , one can solve for $q(j), \forall j \in [0, \kappa]$, independent of β and $f(\cdot)$, which in turn determines (the total revenue from selling $q(j)$ scaled away by that country's aggregator):

$$R(\kappa) \equiv \int_0^\kappa \int_0^{q(j)} r(z) dz dj.$$

Second, the tradable sector of those less productive country y will specialize in export in a trade

equilibrium if the following condition holds:

$$f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) u'(0) < \beta c(0, 0). \quad (30)$$

Note that, when a less productive country y specializes in export, it does not serve any other y country given the symmetry among these countries, and hence it does not import from any of other y countries.

Assuming that the tradable sectors of less productive countries specialize in export, a trading equilibrium among these countries satisfies the following conditions:

$$w f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) u'(0) = \beta c(0, \kappa_y^x); \quad (31)$$

$$f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) u'(0) = w c(0, \kappa_x^y); \quad (32)$$

$$w f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) R(\kappa_y^x) = f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) R(\kappa_x^y); \quad (33)$$

$$f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) u'(0) = c(0, \kappa_x^x). \quad (34)$$

Condition (31) determines the export measure of a y country, condition (32) determines the import measure of such a country, condition (33) says that trade is balanced, and finally condition (34) determines the marginal firm in x country.

Such a trade equilibrium features crowding-in if

$$f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) u'(0) < \beta c(0, \kappa_y^x). \quad (35)$$

When (35) holds, a y country will have a trade sector smaller than κ_y^x under autarky.

Next, we combine conditions (32), (33), and (34) to have

$$R(\kappa_y^x) c(0, \kappa_x^x) = c(0, \kappa_x^y) R(\kappa_x^y). \quad (36)$$

In addition, we combine (31), (32), (34), and (36) into (30), and rewrite the specialization condition as

$$c(0, \kappa_y^x) R(\kappa_x^x) < R(\kappa_x^y) c(0, 0). \quad (37)$$

Finally, we combine (31) and (34) into (32) and rewrite the equilibrium condition for the import

measure of a y country to be

$$f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) u'(0) = \beta \frac{c(0, \kappa_y^x)}{c(0, \kappa_x^x)} c(0, \kappa_x^y) \quad (38)$$

Now, fixing m_x and m_y , for any given $\kappa_y^x \in (0, \infty)$, we can construct $f(\cdot)$ that satisfies Properties (3*) and (4*) and determine β such that a trade equilibrium exists where the tradable sector of each y country specializes in export, has a measure equal to κ_y^x , and features crowding-in. In particular, given $\kappa_x^y \in (0, \infty)$, choose κ_x^y that satisfies (37). It is straightforward to show that $\kappa_x^y > \kappa_y^x$. Given κ_x^y and κ_y^x , we can solve for κ_x^x by (36). Since $\kappa_x^y > \kappa_y^x$, we have $\kappa_x^x > \kappa_x^y$. Given κ_x^x , define $f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj)$ as per condition (34). Given κ_x^y , κ_y^x and κ_x^x , let $f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj)$ be determined by (38). To ensure that

$$f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) > f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj),$$

we choose β such that

$$\beta \frac{c(0, \kappa_y^x)}{c(0, \kappa_x^x)} c(0, \kappa_x^y) > c(0, \kappa_x^x),$$

or

$$\beta > \frac{c(0, \kappa_x^x)^2}{c(0, \kappa_x^y) c(0, \kappa_y^x)}. \quad (39)$$

We are left to show that we can construct $f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj)$ to satisfy (35) and $f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) > f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj)$ (given that $\kappa_x^y > \kappa_y^x$). Given that $\kappa_x^x > c(0, \kappa_x^y)$, it is clearly feasible to define $f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj)$ such that

$$\beta c(0, \kappa_y^x) > f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) > \beta \frac{c(0, \kappa_x^x)}{c(0, \kappa_x^y)} c(0, \kappa_x^y) = f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj).$$

In other words, for any given $\kappa_y^x \in (0, \infty)$, the above procedure allows us to construct, κ_x^y , κ_x^x , $f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj)$, $f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj)$, $f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj)$ that satisfies Property (3*):

$$f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj) > f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z) dz dj) > f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z) dz dj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z) dz dj),$$

and β subject to condition (39), such that $\kappa_y^x, \kappa_x^y, \kappa_x^x$ correspond to a trade equilibrium among x countries and y countries, featuring a crowded-in trade sector in every y country that is at the

same time specialized in export under trade. With $f'(\int_0^{\kappa_y^x} \int_0^{q(j)} u(z)dzdj)$, $f'(m_x \int_0^{\kappa_x^y} \int_0^{q(j)} u(z)dzdj)$, and $f'(m_x \int_0^{\kappa_x^x} \int_0^{q(j)} u(z)dzdj + m_y \int_0^{\kappa_y^x} \int_0^{q(j)} u(z)dzdj)$ thus constructed, it is then straightforward to construct $f(\cdot)$ that satisfies Properties (3*) and (4*). Q.E.D.

Proof of Proposition 19

Let y denote a small country and x a large country. For any m_L and m_S , trade balance of either a large country or a small country requires

$$\int_0^{\kappa_x^y} N_y \frac{w^y}{\lambda^y} \int_0^{q_x^y(j)} r(z)dzdj = \int_0^{\kappa_y^x} N_x \frac{w^x}{\lambda^x} \int_0^{q_y^x(j)} r(z)dzdj.$$

Since

$$\frac{w^y}{\lambda^y} u'(0) = w^x c(0, \kappa_x^y) \quad \text{and} \quad \frac{w^x}{\lambda^x} u'(0) = w^y c(0, \kappa_y^x),$$

the trade balance condition can be rewritten as:

$$\int_0^{\kappa_x^y} N_y \frac{w^x c(0, \kappa_x^y)}{u'(0)} \int_0^{q_x^y(j)} r(z)dzdj = \int_0^{\kappa_y^x} N_x \frac{w^y c(0, \kappa_y^x)}{u'(0)} \int_0^{q_y^x(j)} r(z)dzdj,$$

or

$$\frac{w^x}{w^y} c(0, \kappa_x^y) \int_0^{\kappa_x^y} \int_0^{q_x^y(j)} r(z)dj = \frac{N_x}{N_y} c(0, \kappa_y^x) \int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z)dzdj. \quad (40)$$

Since y is smaller than x ($N_y < N_x$), $w^y > w^x$ as per Proposition 13, hence the left hand side of equation (40), i.e., the revenue of country x exporting to country y , is bounded above:

$$\frac{w^x}{w^y} c(0, \kappa_x^y) \int_0^{\kappa_x^y} \int_0^{q_x^y(j)} r(z)dzdj < c(0, \kappa_y^x) \int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z)dzdj.$$

It is straightforward to verify that $c(\kappa_y^x) \int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z)dzdj$ is increasing in κ_y^x . Therefore, should trade always crowd out less productive firms and hence $\kappa_x^y < \kappa_x^c$ for all $\frac{N_x}{N_y}$, we would have the left hand side of equation (40)

$$c(0, \kappa_y^x) \int_0^{\kappa_x^y} \int_0^{q_x^y(j)} r(z)dzdj < c(0, \kappa_x^c) \int_0^{\kappa_x^c} \int_0^{q_x^c(j)} r(z)dzdj$$

for all $\frac{N_x}{N_y}$. We prove next by contradiction that this cannot be true in equilibrium.

We begin with the observation that the right hand side of equation (40), i.e., the import expense

of country y , is bounded below by

$$c(0,0) \frac{N_x}{N_y} \int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z) dz dj.$$

We can show that this lower bound is unbounded:

$$\lim_{\frac{N_x}{N_y} \rightarrow \infty} \frac{N_x}{N_y} \int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(q) dq dj = \lim_{\frac{N_x}{N_y} \rightarrow \infty} \frac{\int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z) dz dj}{\frac{N_y}{N_x}} = \infty.$$

To see this, note that $\lim_{\frac{N_x}{N_y} \rightarrow \infty} \frac{\int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z) dz dj}{\frac{N_y}{N_x}} = \infty$ if $\lim_{\frac{N_x}{N_y} \rightarrow \infty} \kappa_y^x > 0$. Suppose $\lim_{\frac{N_x}{N_y} \rightarrow \infty} \kappa_y^x = 0$, we apply L'Hopital's rule and differentiate the numerator $\int_0^{\kappa_y^x} \int_0^{q_y^x(j)} r(z) dz dj$ with respect to $\frac{N_x}{N_y}$, and the derivative is

$$\int_0^{q_y^x(\kappa_y^x)} r(z) dz \frac{d\kappa_y^x}{d\frac{N_x}{N_y}} + \int_0^{\kappa_y^x} r(q_y^x(j)) \left(\frac{\partial q_y^x(j)}{\partial \frac{N_x}{N_y}} \right) dj.$$

The first term, $\int_0^{q_y^x(\kappa_y^x)} r(z) dz \frac{d\kappa_y^x}{d\frac{N_x}{N_y}}$, is equal to zero as $q_y^x(\kappa_y^x) = 0$. For the second term, note that $\frac{\partial q_y^x(j)}{\partial \frac{N_x}{N_y}} = 0$ if marginal cost is constant in output.

In this case, we reapply L'Hopital's rule and differentiate $\int_0^{q_y^x(\kappa_y^x)} r(z) dz \frac{d\kappa_y^x}{d\frac{N_x}{N_y}}$ with respect to $\frac{N_x}{N_y}$, and the derivative is

$$u'(0) \frac{\partial q_y^x(\kappa_y^x)}{\partial \kappa_y^x} \left(\frac{d\kappa_y^x}{d\frac{N_x}{N_y}} \right)^2 > 0,$$

since $\frac{\partial q_y^x(\kappa_y^x)}{\partial \kappa_y^x} > 0$ and $\frac{d\kappa_y^x}{d\frac{N_x}{N_y}} \neq 0$.

Since the left hand side of equation (40) is bounded above by a finite number should trade always crowd out less productive firms, the trade balance condition cannot hold. Contradiction.

Therefore, for any (m_L, m_S) , there must exist $\hat{N}(m_L, m_S)$ such that trade indeed crowds in less productive firms in large countries provided that a large country is $N > \hat{N}(m_L, m_S)$ times larger than a small country. Q.E.D.