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The impacts of Net Stable Funding Ratio requirement on Banks' choices of debt maturity

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ABSTRACT

In this paper, we study the impacts of the Net Stable Funding Ratio (NSFR) requirement on banks' choices of debt maturity and asset structures, with consequences for banks' profitability and social welfare. We develop a model in which the manager of a bank determines both debt maturity structure (short-term vs. long-term debt) and asset composition (cash vs. risky assets). To address the incongruence of goals between the bank manager and the bank stakeholders, in our model we assume that the manager receives only a proportion of the bank's profit in her pay schedule. We demonstrate that the optimal choices of the manager regarding debt maturity and asset structure lead to socially inefficient (second-best) outcomes because the manager internalizes only part of the social benefit. We then study the implications of the NSFR requirement on the manager's choices and demonstrate that the NSFR requirement can enhance social welfare and reach an efficient (first-best) outcome, if a sufficiently low weight of short-term debt as available stable funding is required by regulation. Further, we find that under the same conditions the NSFR requirement reduces banks' use of short-term financing and thus increases the probability of banks' survival and profits from the *ex ante* point of view, while it decreases banks' profits from the *ex post* point of view, since it reduces the threshold for banks' survival. Our main results have some interesting empirical implications: under certain conditions, the NSFR requirement may reduce both bank failures and banks' observed profits.

1. Introduction

The reliance on short-term financing and the maturity mismatch problem it caused was cited as a major feature of the global financial crisis from 2007 to 2009 (Brunnermeier, 2009; Diamond and Rajan, 2009; Hellwig, 2009). To address this maturity mismatch problem in banking sector, new regulatory rules were introduced after the crisis. The Net Stable Funding Ratio (NSFR) requirement, which aims to reduce the heavy reliance on short-term funding by banks, is an essential part of the Basel III reforms (Basel Committee on Banking Supervision (2011, 2014)). The NSFR is equal to banks' available stable funding divided by banks' potential funding needs, and is required to be larger than 100% to ensure that banks are not exposed to excessive liquidity risk. The NSFR requirement, which is to be implemented in 2018, constitutes a major part of the liquidity regulations in Basel III.

Research on this new regulation is much needed. Some recent works such as King (2013) and Dietrich et al. (2014) study empirically whether the banks fulfilled the NSFR requirement before 2010, and both find that most of the banks did not.[1] However, research based on historical data may have limited predictive power about the effect of the NSFR requirement, since the final version of NSFR was presented by Basel Committee on Banking Supervision (2014), and its implementation is supposed to be completed until 2018. In contrast to those studies, in this paper we develop a theoretical framework to evaluate how the NSFR requirement could change the banks' use of short-term debt, and influence banks' (ex ante and ex post) profits, and more importantly, social welfare.

To answer these questions, we first study the choices of the bank manager regarding asset composition and debt maturity structure before the NSFR is enforced. To address the incongruence

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[1] They also assessed how this regulation could affect banks' risk and profitability, but obtained different results. King (2013) shows that banks' profitability will be reduced since risk is lower under regulation, while (Dietrich et al., 2014) do not find notable evidence for this.

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of goals between the bank manager and the bank stakeholders, in our model we assume that the manager receives only a proportion of the bank's profit in her pay schedule. Taking into consideration that creditors must break even, the manager chooses the asset composition and maturity structure at the same time to maximize expected payoff. We demonstrate that the optimal choices of the manager regarding debt maturity and asset structure lead to socially inefficient (second-best) outcomes because the manager internalizes only part of the social benefit. We find an interior solution for the optimal debt maturity structure chosen by the manager, i.e., both short-term and long term debts will be issued by the bank. Since the manager receives only a portion of banks' profits in her pay schedule, the manager's decision leads towards a higher proportion of short-term debt than what social optimum would entail. Thus without the NSFR regulation, the banking equilibrium could cause a social welfare loss. The incongruence of goals between the bank owner and the bank manager cannot be resolved directly as long as the manager does not benefit from the whole profit of the bank. Thus regulation is needed to restore social welfare.

Next we study the implications of the NSFR requirement on the manager's choices and demonstrate that the NSFR requirement can enhance social welfare and reach an efficient (first-best) outcome under certain conditions. We find that as long as the regulation rule is strict enough, i.e., it endows a low enough weight for short-term debt as available stable funding, the NSFR requirement can reduce the banks' use of short-term debt. Thus this regulation can indeed deal with the maturity mismatch problem and increase social welfare. However, our results also show that regulators must be cautious in the sense that if the regulation rule is not strict enough about the short-term debt, it may lead the manager to choose an improper combination of asset composition and debt maturity structure, which reduces social welfare. As for banks' profitability, we examine the effect from both ex ante and ex post points of view, where banks' ex ante profit is the banks' expected profits as seen from the initial date, which is the normal measure of banks' profits, and ex post profit is the expected profits conditional on banks' survival. Our model predicts that the NSFR requirement increases the probability of banks' survival and profits from the ex ante point of view, because it alleviates the goal-incongruence problem between the bank and the manager; while it reduces banks' profits from the ex post point of view, it also reduces the threshold for survival. Our main results have some interesting empirical implications: under certain conditions, the NSFR requirement may reduce both bank failures and banks' observed profits.

1.1. Net Stable Funding Ratio

After the 2007–2009 financial crisis, liquidity risk has become a main concern of both researchers and regulators. Specifically, Basel III introduces the Net Stable Funding Ratio (NSFR) requirement on the liability side of the banks. The NSFR is defined as:

$$NSFR = \frac{Available\,Stable\,Funding\,(ASF)}{Required\,Stable\,Funding(RSF)} = \frac{\sum_i ASF\,factor_i \times Liablities_i}{\sum_j RSF\,factor_j \times Assets_i} \tag{1}$$

where ASF is calculated as the weighted sum of the bank's capital and liabilities, which are divided into five categories with different weights (ASF factors) according to their funding stability. RSF is calculated as the weighted sum of the bank's assets, which are divided into seven categories with different weights (RSF factors) according to their exposures to liquidity risk. Intuitively, ASF is the stable funding that will not be withdrawn in the short-term and RSF represents the required funding of the bank related to the liquidity risk. The NSFR ratio is required to be no less than 100% and is designed "to limit over-reliance on short-term wholesale funding during times of buoyant market liquidity and encourage better assessment of liquidity risk across all on- and off-balance sheet items"[2].

1.2. Related literature

Our paper is related to the current literature on debt maturity structures. Similar to our paper, Eisenbach (2013) allows the maturity structure to be observable. The author endogenizes the asset's liquidation value so that some banks can sell their assets at a price higher than the assets' true value, thus early liquidation needs not to be inefficient if there is no aggregate risk. Our paper focuses on the early liquidation cost brought by roll-over risk induced by short-term debt, which allows us to discuss welfare improvement by regulatory devices. In Brunnermeier and Oehmke (2013), the authors assume that the debt maturity structure is unobservable and analyze the coordination problem among creditors. The negative externalities exerted by the short-term debt holders on the long-term debt holders will cause the rat race of debt maturity. Their model leads to a corner solution: the bank chooses to issue either only short-term debt or only long-term debt. Thus in their model one cannot discuss how the maturity structure reacts to the environment. In our model the bank chooses to issue both long-term and short-term debts, and therefore our framework can be used to study the optimal choice of debt maturity structure.

In their recent work, Diamond and He (2014) consider a model in which long-term and short-term debts differentially affect the wealth transfer to debt holders and focus on the problem of debt overhang. Huberman and Repullo (2013) develop a model to study how the composition of debt with different maturities affects the equity holder's incentive of risk-shifting. In contrast to these recent studies, in this paper we construct a model to allow the bank manager to determine both the bank's debt structure and asset allocation. We study the manager's optimal choices both on the liability side and the asset side of the bank's balance sheet.

Our paper also supplements the literature on the NSFR requirement. (King, 2013) estimates the NSFR for banks from 15 countries and Dietrich et al. (2014) examine whether Western European banks from 1996 to 2010 have fulfilled the NSFR requirement. However, since the final structure of NSFR emerged in 2014, empirical research based on historical data has limited predictive power for banks' behaviors after the requirement comes into effect. In this paper, we propose a theoretical framework to evaluate the impacts of the NSFR requirement on banks' choices. We also study the welfare consequences of the regulation at the theoretical frontier.

The rest of this paper is organized as follows: in Section 2 we present the model without the NSFR requirement and analyze the manager's maximization problem; in Section 3 we derive the optimal asset composition and debt maturity structure of the bank's manager; social optimum is also shown in this section; in Section 4 we study the impacts of the NSFR requirement on the manager's choices, bank profitability, and social welfare; in Section 5 we discuss the relaxation of some assumptions and the robustness of our results; and in Section 6 we conclude.

2. The model without NSFR requirement

2.1. Model setup

We consider a model with three dates $t = 0, 1, 2,$ as shown in Fig. 1. At date 0, a bank hires a manager to run the bank. The man-

[2] See "Basel III: A global regulatory framework for more resilient banks and banking systems" (2010), p. 17.

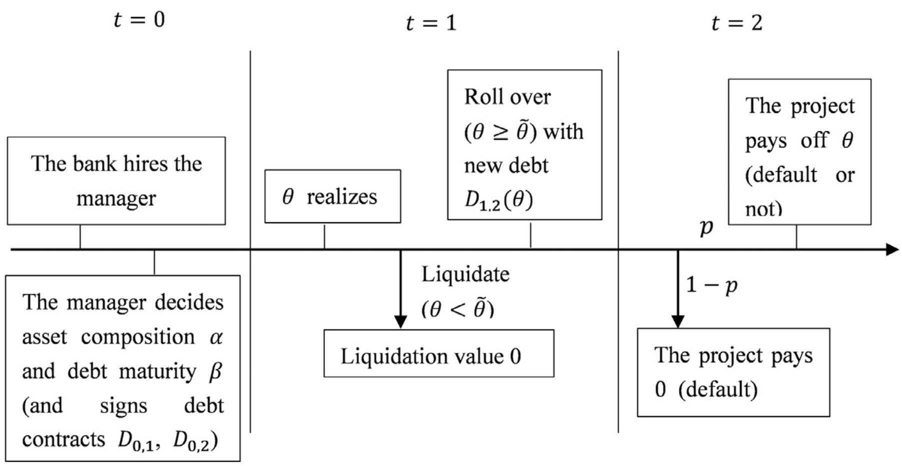

Fig. 1. Timeline of our model.

ager gets a performance pay at date 2, i.e., she receives $s \in (0, 1)$ share of the bank's profit. To earn her payment, for each operating period, the manager has to make efforts to manage the bank, which induces disutility $k > 0$ for her per period. As it will be shown later, the social loss of inefficiency in our model is caused by the incongruence of goals between the bank manager and the bank owner (stakeholders).[3] The manager has to make investment and financing decisions for the bank at date 0. The bank can invest in two kinds of assets: a riskless asset and a risky asset. Both kinds of assets require initial cost normalized to 1. Risk-free interest rate is set to be 0 for simplicity, and the riskless asset can be interpreted as cash in this model. The risky asset pays off at date 2, with no cash flow generated at date 1. The payoff V can take two possible values: $\theta > 0$ with probability $p \in (0, 1]$ and 0 with probability $1 - p$. We assume θ can not be observed until date 1, and has a uniform distribution over $[0, \bar{\theta}]$ as viewed from date 0, i.e., the probability density function is $f(\theta) = 1/\bar{\theta}$. Positive NPV condition for the risky asset is thus $p\bar{\theta} > 2$. Apparently θ is an indicator of the asset's fundamental value ($\mathbb{E}(V \mid \theta) = p\theta$), and can be used as a signal of the asset value at date 1.[4] We assume that both kinds of asset have constant returns to scale, so we can normalize the total investment value to 1 unit, with $\alpha \in [0, 1]$ proportion of risky asset and $1 - \alpha$ proportion of cash. We call this α as the bank's *asset composition*.

Now we turn to the banker's financing problem. We assume that the bank has no initial capital, and she has to finance the total investment. The main results of our model will not be changed if the bank has some initial equity and we will discuss the effect of leverage ratio requirement in Section 5. For financing, the bank can issue debt (such as bank debenture) to investors. There are a continuum of homogeneous investors (with total measure larger than 2). Each risk neutral creditor is endowed with 1 unit of capital, so the bank has to borrow from multiple creditors. At date 0, the bank can issue a short-term or long-term debt contract. For simplicity, all the short-term (or long-term) contracts are the same (have the same face value). Short-term debt matures at date 1 with face value $D_{0,1}$; long-term debt matures at date 2 with face value $D_{0,2}$. Denote the proportion of short-term creditors as $\beta \in [0, 1]$, which is called the *debt maturity structure* of the bank. We allow the bank's maturity structure to be observed to all creditors. This is commonly seen in reality and adopted by Diamond (1991) and Eisenbach (2013).

At the beginning of date 1, the payoff θ of the risky asset is realized. After that, the short-term debt matures and the manager has to roll over by issuing new short-term debts (from date 1 to date 2) to new creditors. If new short-term debt contracts are accepted by creditors, the bank's investment goes on, and we call the new creditors as roll-over creditors. We denote the face value of a roll-over debt as $D_{1,2}(\theta)$. Note that this face value is related to θ, because roll-over creditors decide whether or not to lend based the information about θ at date 1. If creditors refuse to lend to the bank at date 1, the bank's risky investment has to be liquidated prematurely. The liquidation value will be smaller than the expected value of the asset, which means that there is a liquidation cost. To simplify our analysis, we assume the liquidation value to be zero.[5] We also assume that long-term creditors and short-term creditors equally split the value of the bank by initial costs at date 1 if the risky asset is early liquidated at date 1, and by face values if the bank defaults at date 2.

At date 2, all uncertainty resolves and all debts get paid back or go default. If the asset value is large enough, the bank earns a positive profit of π and the manager gets a performance pay of $s\pi$.

2.2. The break-even conditions

We assume that all creditors must break even. In fact, the face values of short-term and long-term debts at date 0 ($D_{0,1}$ and $D_{0,2}$) and roll-over debt at date 1 ($D_{1,2}(\theta)$) are all related to asset composition and maturity structure. Thus before we specify the manager's problem at date 0, we have to calculate face values of all debts given α and β. By backward induction, we need first to solve the face value of roll-over debt $D_{1,2}(\theta; \alpha, \beta)$ issued at date 1. At the interim date, roll-over creditors are willing to accept this face value only if they can break even. The cost per unit of roll-over debt for creditors is the face value of the short-term debt issued at date 0, i.e., $D_{0,1}$; while the benefit of the roll-over debt is uncertain: it can be $D_{1,2}(\theta; \alpha, \beta)$ paid back or a part of remaining value of the bank's value if it goes bankrupt. Given the face values of the debts issued at date 0 ($D_{0,1}$ and $D_{0,2}$), the asset composition α, debt maturity structure β, and potential positive payoff θ, for

[3] Our model can be embedded into a principle-agent framework with asymmetric information where in order to resolve the moral hazard problem, the manager (as the agent) is given an incentive payment, leading towards a second-best outcome. To simplify the analysis, in our model we allow the manager to be risk neutral and be willing to spend effort.

[4] Actually, θ also affects the variance of the asset. However, we will show in Section (5.1) that only the expected asset value matters in our model, which enables us to focus on θ for its role as the indicator of fundamental value.

[5] It can be shown (in Section 5.2) that adopting the alternative assumption with the liquidation value as a positive fraction $\lambda \in (0, 1)$ of the expected value of the investment will not change the main results of this paper

any face value $D_{1,2}(\theta;\ \alpha,\ \beta)$ at date 1, there could be two possible situations for the roll-over creditors:

1. If $\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}>\alpha\theta+(1-\alpha)$, the bank's total asset value can not cover the total debt cost even if the risky asset pays θ, so the bank is certain to default at date 2, and roll-over creditors will split the investment payoff with long-term creditors by face values. Thus the roll-over break-even condition for this case is:

$$D_{0,1}=\frac{D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}(\alpha p\theta+(1-\alpha)) \tag{2}$$

where $\alpha p\theta+(1-\alpha)$ is the bank value expected at date 1, and $\frac{D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}$ is the proportion that one unit of roll-over debt receives ($\frac{\beta D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}$ is the share of all roll-over creditors). If we let $D_{1,2}(\theta;\ \alpha,\ \beta)$ go into infinity in this equation, we can get the lower bound of θ which can make this equation hold, denoted as $\tilde{\theta}(\alpha,\beta)\triangleq\frac{\beta D_{0,1}-(1-\alpha)}{\alpha p}$; and if we let $D_{1,2}(\theta;\alpha,\beta)=\frac{\alpha\theta+(1-\alpha)-(1-\beta)D_{0,2}}{\beta}$ in this equation, we can get the upper bound of θ that can make this equation hold, denoted as $\hat{\theta}(\alpha,\beta)$, i.e., the unique solution to:

$$\frac{(\alpha\theta+(1-\alpha)-(1-\beta)D_{0,2})(\alpha p\theta+(1-\alpha))}{\alpha\theta+(1-\alpha)}=\beta D_{0,1}.$$

Thus the condition for the existence of a solution to Eq. (2) is: $\theta\in(\tilde{\theta}(\alpha,\beta),\hat{\theta}(\alpha,\beta))$.

2. If $\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}\le\alpha\theta+(1-\alpha)$, then all the creditors can get promised face values if the high outcome θ of the investment realizes at date 2 (with probability p), and $D_{1,2}(\theta;\ \alpha,\ \beta)$ is paid back to each unit of roll-over debt. While if the risky asset pays nothing (with probability $1-p$), creditors can only split the remaining cash $(1-\alpha)$, with the same weight as in condition (2). So the roll-over break-even condition is:

$$D_{0,1}=p\cdot D_{1,2}(\theta;\alpha,\beta)+(1-p)\times\frac{D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}(1-\alpha) \tag{3}$$

The condition for there existing a solution to Eq. (3) is: $\hat{\theta}(\alpha,\beta)\le\theta\le\bar{\theta}$.

As long as Eq. (2) or (3) has a solution for $D_{1,2}(\theta;\ \alpha,\ \beta)$, the bank could roll over by issuing roll-over debt with this face value that creditors are willing to accept, and thus the bank is able to roll over if:

$$\theta>\tilde{\theta}(\alpha,\beta). \tag{4}$$

Clearly, this condition is equivalent to $\alpha p\theta+(1-\alpha)>\beta D_{0,1}$, which means that the bank's expected asset value can cover the face values of all short-term debts. This condition doesn't take long-term debt into consideration because although all creditors have equal seniority, roll-over creditors have priority *de facto* because they can always require a high enough face value of $D_{1,2}(\theta;\ \alpha,\ \beta)$ to get as much as the bank's value. In face, if we let $D_{1,2}(\theta;\alpha,\beta)\to+\infty$ in (2), this condition becomes $\beta D_{0,1}=\alpha p\theta+(1-\alpha)$. This implies that as long as the bank's expected asset value exceeds the short-term debt payment, roll-over creditors can always break even by requiring a sufficiently high face value $D_{1,2}(\theta;\ \alpha,\ \beta)$.

Apparently, it is more difficult to roll over as the roll-over creditors' lending cost $\beta D_{0,1}$ increases, the bank's cash holding $1-\alpha$ decreases and the probability of risky asset's positive payment p decreases. Note that if $\beta D_{0,1}\geqslant p\alpha\bar{\theta}+1-\alpha$, the short-term debt will never be rolled over and $\tilde{\theta}(\alpha,\beta)=\overline{\theta}$, and if $\frac{(\alpha\theta+(1-\alpha)-(1-\beta)D_{0,2})(\alpha p\theta+(1-\alpha))}{\alpha\theta+(1-\alpha)}<\beta D_{0,1}$, the bank will certainly go default and $\hat{\theta}(\alpha,\beta)=\overline{\theta}$.

With roll-over debt at date 1 specified, we can now get back to debts at date 0. For short-term creditors, they lend 1 unit of cash to the bank at date 0, expecting debt repayment at date 1. There is only one sort of risk for them: the roll-over risk. If the realization θ at date 1 is too low ($\theta<\tilde{\theta}(\alpha,\beta)$), then short-term debts will not be rolled over, the risky asset will be early liquidated and all creditors equally split the cash held by the bank Note that long-term debt doesn't mature at date 1, so short-term and long-term creditors have to split the cash by what they paid at date 0, i.e., equal split among all creditors. Short-term creditors get $\beta(1-\alpha)$ in total and each short-term creditor gets $1-\alpha$. If the realization θ is high enough ($\theta>\tilde{\theta}(\alpha,\beta)$), the debt will be rolled over and the short-term debt can get the face value $D_{0,1}$ fully paid back. The short-term creditors can break even if the expected payment of the debt could cover the cost of the debt:

$$(1-\alpha)\int_0^{\tilde{\theta}(\alpha,\beta)}f(\theta)d\theta+D_{0,1}\int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}}f(\theta)d\theta=1 \tag{5}$$

As for long-term creditors, they have to consider three sources of risks after lending to the bank at date 0: the first is the roll-over risk that if $\theta<\tilde{\theta}(\alpha,\beta)$ is at date 1, a long-term creditor can only receive $1-\alpha$, which is exactly the same with the risk of short-term creditors. The second is default risk at date 2. If the realization θ is large enough for the debt to roll over, but not large enough for both kinds of creditors to get fully paid, i.e., $\tilde{\theta}(\alpha,\beta)<\theta<\hat{\theta}(\alpha,\beta)$, roll-over creditors will require a very high face value, which makes the bank certain to default at date 2, and creditors have to split the default value by face values. A long-term creditor can split $\frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}$ share of bank's expected asset value $\alpha p\theta+(1-\alpha)$. And at last, even if $\theta\ge\hat{\theta}(\alpha,\beta)$ is learned at date 1, only if θ is realized with probability p at date 2 can a long-term creditor get full payment $D_{0,2}$. Otherwise she can only split $\frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}$ proportion of the remaining cash $1-\alpha$. The extra risks of long-term creditors imply that although long-term creditors have the same seniority as short-term creditors, they may suffer more because they can not re-negotiate with the bank about the face values of the debts. Taking all the risks into consideration, the break-even condition for long-term creditors is:

$$\begin{cases}(1-\alpha)\int_0^{\tilde{\theta}(\alpha,\beta)}f(\theta)d\theta\\+\int_{\tilde{\theta}(\alpha,\beta)}^{\hat{\theta}(\alpha,\beta)}\frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}(\alpha p\theta+(1-\alpha))f(\theta)d\theta\\ \int_{\hat{\theta}(\alpha,\beta)}^{\bar{\theta}}\left(p\cdot D_{0,2}+(1-p)\frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}(1-\alpha)\right)f(\theta)d\theta\quad=1\end{cases} \tag{6}$$

Up to now, we have specified all the break-even conditions (2), (3), (5), (6), and we know that given the existence of all other face values, there is always a non-negative face value $D_{1,2}(\theta;\ \alpha,\ \beta)$ such that roll-over creditors break even once $\theta>\tilde{\theta}(\alpha,\beta)$ as in Eq. (4), so we only have to focus on the break-even conditions at date 0. It is easy to check that $(\alpha,\beta)=(0,0)$ induces no roll-over risk and all break-even conditions have solutions. However, it is possible that for some asset composition and debt maturity structure, the break even conditions (5) and (6) may not be both satisfied, so it is necessary to define feasible asset composition and maturity structures.

Definition 1. A pair of structures $(\alpha,\ \beta)$ is feasible, if there exists non-negative face values $D_{0,1}$, $D_{0,2}$ such that short-term and long-term creditors can both break even at date 0.

2.3. The manager's problem

After all debt face values are determined, we can discuss the bank's expected profit. For a given feasible pair of structures (α, β), and corresponding face values $\{D_{0,1}(\alpha, \beta), D_{0,2}(\alpha, \beta), D_{1,2}(\theta; \alpha, \beta)\}$, the profit from the bank's investment at date 2 can be written as follows:

$$\pi(\alpha, \beta) = \int_{\hat{\theta}(\alpha,\beta)}^{\bar{\theta}} p(\alpha\theta + (1-\alpha) - \beta D_{1,2}(\theta; \alpha, \beta) \\ -(1-\beta)D_{0,2}(\alpha, \beta))f(\theta)d\theta$$

The profit for the bank at date 2 is its total asset value $\alpha\theta + (1-\alpha)$ minus final debt payments $\beta D_{1,2}(\theta; \alpha, \beta) + (1-\beta)D_{0,2}(\alpha, \beta)$. The integral starting from $\hat{\theta}(\alpha, \beta)$ is because for the bank to earn positive profits from the investment, it needs not only short-term debt rolled over at date 1, but also low enough face values such that the realized high payoff of the investment plus cash holding could cover. In addition, θ has to be realized (in probability p) at date 2 for the bank to earn some profits. In fact, we can rewrite the bank's profit in a more intuitive way. Using break-even conditions (2), (3), (5), (6), we can rewrite the profit function as:

$$\pi(\alpha, \beta) = \alpha\left(\int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} p\theta f(\theta)d\theta - 1\right) \tag{7}$$

Since all creditors break even, the bank's profit is just the total net benefit of all investments. And cash generates zero return, the net benefit of all investments is solely the net benefit of the risky asset (α proportion in total investment). The total cost for one unit of risky asset is the starting cost 1, and the benefit of the investment is its expected payoff, which is 0 if the asset is early liquidated $\theta < \tilde{\theta}(\alpha, \beta)$and is $p\theta$ if it continues at date 1 $\theta > \tilde{\theta}(\alpha, \beta)$.

With the profit of the bank specified, we can calculate the manager's payoff. Manager gets s share of the bank's profit, and suffers from disutility for certain at date 1, and also at date 2 if the bank lasts to that date ($\theta > \tilde{\theta}(\alpha, \beta)$). Thus her payoff can be written as:

$$M(\alpha, \beta) = s\pi(\alpha, \beta) - k\left(1 + \int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} f(\theta)d\theta\right). \tag{8}$$

We can also derive social by adding all agents' net payoffs in the economy. Since all creditors break even and has zero net payoff, we only have to consider the bank's net profit $(1-s)\pi(\alpha, \beta)$ and manager's payoff $M(\alpha, \beta)$:

$$W(\alpha, \beta) = (1-s)\pi(\alpha, \beta) + M(\alpha, \beta) \\ = \pi(\alpha, \beta) - k\left(1 + \int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} f(\theta)d\theta\right). \tag{9}$$

Apparently, without regulation the manager will choose a different pair of (α, β) from the social optimal one. It is due to the incongruence of goals between the bank manager and the bank stakeholders: the manager bears the whole cost of the economy while getting only s share of the benefit.

3. The asset composition and debt maturity choices without NSFR requirement

3.1. Manager'S optimal choices

In this subsection, we derive the manager's optimal asset composition and maturity structure choices without NSFR regulation. To confirm that the manager's maximization problem is well-defined, we have to find the conditions such that all pairs of structures (α, β) are feasible. Note that if $1-\alpha \geq \beta D_{0,1}(\alpha, \beta)$, the bank has enough cash to pay the short-term debt at date 1 and there is no risk of early liquidation. Thus short-term debt at date 0 is riskless and $\tilde{\theta}(\alpha, \beta)$ must be 0. Apparently, all pairs of structures (α, β) satisfying this inequality have no roll-over risk and are feasible. We can see from expression (8) that with $\tilde{\theta}(\alpha, \beta) = 0$, and the manager's objective function becomes:

$$M(\alpha, \beta) = s\alpha\left(\int_0^{\bar{\theta}} p\theta f(\theta)d\theta - 1\right) - 2k.$$

However, if $1-\alpha > \beta D_{0,1}(\alpha, \beta)$, the cash holding can not cover all the short-term debt payment at date 1. Then there is roll-over risk and $\tilde{\theta}(\alpha, \beta)$ is positive. In this case, we have to find conditions for the feasibility of (α, β) pairs. The following lemma shows the condition under with there exists solutions to break-even conditions both (5) and (6).

Assumption A. $p\bar{\theta} > 4$.

Lemma 1. *Under Assumption A, there exists positive face values $D_{0,1}$ and $D_{0,2}$ such that short-term and long-term creditors can both break even for all asset composition and maturity structure (α, β) ∈ [0, 1] × [0.1] at date 0.*

Assumption A is stronger than the positive NPV condition $p\bar{\theta} > 2$, because there is asset value loss due to early liquidation: the expected net asset value has to be larger than 0 to support all pairs of structures (α, β). In the proof of Lemma 1 (see Appendix), we show that there are two solutions $D_{0,1}$ to break-even condition (5) for each pair of (α, β). However, the larger solution will go to infinity as β approaches 0, which obviously violates the intuition that when there is less roll-over risk, the face value of short-term debt should be smaller. For this reason, we pick the smaller one as the solution, and explicitly express the face value of short-term debt at date 0 as:

$$D_{0,1}(\alpha, \beta) = \frac{2\left(\alpha p\bar{\theta} + (1-\alpha)^2\right)}{(1+\beta)(1-\alpha) + \alpha p\bar{\theta} + \sqrt{\left((1+\beta)(1-\alpha) + \alpha p\bar{\theta}\right)^2 - 4\beta\left(\alpha p\bar{\theta} + (1-\alpha)^2\right)}}. \tag{10}$$

Using this expression, we can characterize the combination of (α, β) such that $1-\alpha > \beta D_{0,1}(\alpha, \beta)$ holds.

Lemma 2. *Under Assumption A, the bank will suffer from positive roll-over risk at date 1 if the following inequality holds:*

$$\beta > \beta^b(\alpha) \triangleq 1-\alpha. \tag{11}$$

The result of Lemma 2 is quite intuitive: if inequality (11) holds, the cash holding can never cover the whole short-term debt payments ($\beta D_{0,1} \geq \beta > 1-\alpha$) at date 1, even if short-term debt is risk-free. In this case, it is always possible that θ is so low ($\beta D_{0,1} > \alpha\theta + (1-\alpha)$, or $\theta < \tilde{\theta}$) that the bank fails to roll over. Only if θ is high enough ($\beta D_{0,1} < \alpha\theta + (1-\alpha)$, or $\theta > \tilde{\theta}$) can the bank get new funding. From expression (10), we can derive the corresponding cut-off value of θ for roll-over:

$$\tilde{\theta}(\alpha,\beta)=\frac{\alpha p\bar{\theta}-(1-\alpha)(1-\beta)-\sqrt{\left((1+\beta)(1-\alpha)+\alpha p\bar{\theta}\right)^2-4\beta\left(\alpha p\bar{\theta}+(1-\alpha)^2\right)}}{2\alpha p}. \quad (12)$$

We can see that this cut-off value increases as the expected value of the risky asset $p\bar{\theta}/2$ decreases. More importantly, we can further find that as the bank uses more short-term debt, or invests more in risky asset, the short-term creditors will require a higher face value, which induces more roll-over risk, or a higher $\tilde{\theta}(\alpha,\beta)$. We show this in the following lemma.

Lemma 3. *If $\alpha+\beta>1$, as more short-term debt is used or more risky asset is invested at date 0, it is more difficult for the bank to roll over at date 1, i.e., $\frac{\partial\tilde{\theta}(\alpha,\beta)}{\partial\alpha}\geq 0$ and $\frac{\partial\tilde{\theta}(\alpha,\beta)}{\partial\beta}>0$.*

Now we know that under Assumption A, all maturity structures are feasible. Based on this, we can properly derive the manager's optimal choices. There are two variables α and β the manager has to decide. To solve the optimum more intuitively, we can first derive the optimal debt maturity structure β given asset composition α, and first order conditions for β is:

$$\frac{\partial M(\alpha,\beta)}{\partial\beta}=\left(k-s\alpha p\tilde{\theta}(\alpha,\beta)\right)Q_\beta(\alpha,\beta), \quad (13)$$

where $Q(\alpha,\beta)\triangleq\tilde{\theta}(\alpha,\beta)/\bar{\theta}$ is the probability that the bank's risky asset will be early liquidated.

We know that given α, if $\beta\in[0,1-\alpha]$, then $\tilde{\theta}=0$ and $\frac{\partial M(\alpha,\beta)}{\partial\beta}=0$ from Lemma 2; and if $\beta\in(1-\alpha,1]$, then $\tilde{\theta}>0$ and $\frac{\partial\tilde{\theta}(\alpha,\beta)}{\partial\beta}>0$ from Lemma 3. In the latter case, we need to discuss the derivative in (14) further. From this expression, we can see how the liquidation cost and management cost affect the optimal maturity structure decision of the manager when there is positive roll-over risk. At date 0, as the bank uses more short-term debt, roll-over creditors will require a higher face value and thus a higher fundamental value θ of the risky asset to refinance the debt at date 1(from Lemma 3), which increases the probability of early liquidation, i.e., $Q_\beta(\alpha,\beta)>0$. This generates both benefit (the manager could avoid management cost with a higher probability, as the marginal benefit is expressed by $kQ_\beta(\alpha,\beta)$) and cost (the bank suffers liquidation cost with a higher probability and faces higher debt face values, as the marginal cost is expressed by $s\alpha p\tilde{\theta}(\alpha,\beta)Q_\beta(\alpha,\beta)$, where $\alpha p\tilde{\theta}(\alpha,\beta)$ is the bank's loss from early liquidation and the manager bears s share of it) to the manager. We can see that the benefit depends only on the probability of early liquidation, while the cost also varies with the loss of liquidation value of the risky asset, which is increasing in $\tilde{\theta}(\alpha,\beta)$. In sum, the fundamental trade-off of using short-term debt for manager is between avoiding management cost and reducing liquidation cost. In addition, the cost of short-term debt increases relatively faster than the benefit with β, which makes interior solution possible. Formally, we solve the optimal maturity structure given asset composition in the following lemma.

Assumption B. $k\left(1+\sqrt{1-\frac{4}{p\bar{\theta}}}\right)<2s.$

Lemma 4. *Under Assumption B, given any asset composition α, the optimal debt maturity structure for the manager is:*

$$\beta^*(\alpha)=\begin{cases}1 & \alpha\leq\alpha_a,\\ \frac{\left(\frac{k}{s}+(1-\alpha)\right)\left(\alpha p\bar{\theta}-\frac{k}{s}\right)}{\alpha p\bar{\theta}-\frac{k}{s}(1-\alpha)} & \alpha\geq\alpha_a,\end{cases} \quad (14)$$

and $\beta^*(\alpha)$ is concave and strictly decreasing in α for $\alpha\geq\alpha_a\triangleq\frac{k}{2s}(1+\sqrt{1-\frac{4}{p\bar{\theta}}})$.

This lemma shows that the manager will optimally choose (α, β) as represented by $\beta=\beta^*(\alpha)$ in the region [0, 1] × [0.1]. The property of $\beta^*(\alpha)$ also reveals a basic trade-off for the manager to choose asset composition and asset structure: if the manager wants to increase the proportion of risky asset to raise the profit, she has to reduce the use of short-term debt in order to avoid excessive roll-over risk. With the expression of $\beta^*(\alpha)$, we can easily maximize the manager's payoff by choosing α. The first order condition is

$$\frac{dM(\alpha,\beta^*(\alpha))}{d\alpha}=\left[k-s\alpha p\tilde{\theta}(\alpha,\beta^*(\alpha))\right]Q_\alpha(\alpha,\beta^*(\alpha))+s\left(\int_{\tilde{\theta}(\alpha,\beta^*(\alpha))}^{\bar{\theta}}p\theta f(\theta)d\theta-1\right). \quad (15)$$

Note that a term $\left[k-s\alpha p\tilde{\theta}(\alpha,\beta^*(\alpha))\right]Q_\beta(\alpha,\beta^*(\alpha))\frac{d\beta^*(\alpha)}{d\alpha}$ is omitted in (15) because it equals 0 with $\beta=\beta^*(\alpha)$. We can also understand from the above derivative that the basic trade-off of investing in risky asset for the manager is as follows: a higher proportion of risky asset generates higher expected return and thus a higher payment from the bank (the second term), while a higher proportion of risky assets also raises the roll-over risk, and induces the similar benefit and cost considerations as with choosing the debt maturity structure (the first term). We know that the final effect of debt maturity in optimality is non-negative i.e., $k\geq s\alpha p\tilde{\theta}(\alpha,\beta^*(\alpha))$ from the proof of Lemma 4 and $\int_{\tilde{\theta}(\alpha,\beta^*(\alpha))}^{\bar{\theta}}p\theta f(\theta)d\theta-1>0$ from the proof of Lemma 1. These conditions yield $\frac{dM(\alpha,\beta^*(\alpha))}{d\alpha}>0$. Thus we can easily derive the optimal asset composition and maturity structure for the manager.

Proposition 1. *Under Assumptions A and B, all the pairs of asset composition and maturity structures (α, β) ∈ [0, 1] × [0.1] are feasible, and there is a unique optimal pair of structures (α^*, β^*) for the manager:*

$$\alpha^*=1,\quad\beta^*=\frac{k}{s}\left(1-\frac{k}{s}\frac{1}{p\bar{\theta}}\right). \quad (16)$$

We show the results of Proposition 1 in Fig. 2, with parameters $p=0.5$, $\bar{\theta}=12$, $k=0.1$, $s=0.12$. It can be seen that given asset composition α, the debt maturity structure chosen by the manager $\beta^*(\alpha)$ induces positive roll-over risk (higher than the bound of positive roll-over risk $\beta^b(\alpha)$). Manager chooses $\beta^*(\alpha)=1$ for $\alpha\leq\alpha_a$ and it strictly decreases with α for $\alpha>\alpha_a$. The pair of structures (α^*, β^*) chosen by the manager satisfies $\alpha^*=1$ and $\beta^*=\beta^*(\alpha^*)$.

3.2. Social optimal asset composition and maturity structure

Comparing the social welfare function and the manager's objective function, the only difference is that the social welfare function includes the whole profit of the bank's assets; while the manager only internalizes s share of the it, which can be shown in the following condition:

$$\frac{\partial W(\alpha,\beta)}{\partial\beta}=\left(k-\alpha p\tilde{\theta}(\alpha,\beta)\right)Q_\beta(\alpha,\beta).$$

For the interior solution, FOC of social planner's problem yields $k=\alpha p\tilde{\theta}(\alpha,\beta)$, which is quite intuitive: at date 1, the social cost for the risky asset to continue is the disutility k from the manager; and the social benefit is the asset's total expected value $\alpha p\theta$.

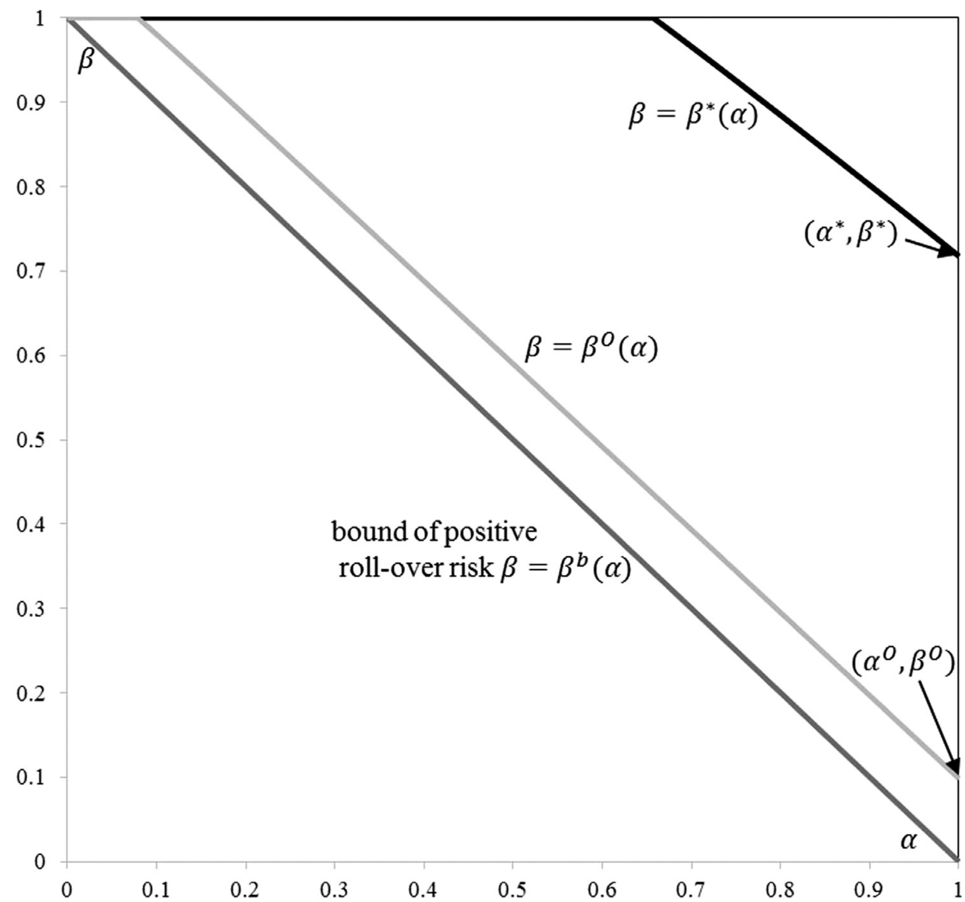

Fig. 2. Asset composition and maturity structure chosen by the manager and social optimum.

So the social optimal rule for the bank to keep the risky asset should be $\theta > \frac{k}{\alpha p}$. We have shown the roll-over rule of creditors is $\theta > \tilde{\theta}(\alpha, \beta)$. Thus when $\tilde{\theta}(\alpha, \beta) = \frac{k}{\alpha p}$, or $k = \alpha p\tilde{\theta}(\alpha, \beta)$, all socially beneficial assets are kept and other assets are liquidated at date 1, which restores the social welfare optimum. As a matter of fact, socially optimal maturity structure implements an optimal rule of roll-over.

The maturity structure chosen by the manger is given by $k = \alpha sp\tilde{\theta}(\alpha, \beta)$, which leads to a $\tilde{\theta}(\alpha, \beta) = \frac{k}{\alpha sp} > \frac{k}{\alpha p}$. It allows some risky assets with negative net social benefit to be kept by the bank at date 1, and thus induces inefficiency. In the inefficient outcome, the manager is less sensitive to early liquidation cost, and is more willing to take roll-over risk. The following proposition shows this result.

Assumption B'. $k\left(1 + \sqrt{1 - \frac{4}{p\bar{\theta}}}\right) < 2.$

Proposition 2. *Under Assumptions A and B', the social optimal debt maturity structure given asset composition is:*

$$\beta^o(\alpha) = \begin{cases} 1 & \alpha \le \frac{k}{2}\left(1 + \sqrt{1 - \frac{4}{p\bar{\theta}}}\right), \\ \frac{(k+(1-\alpha))(\alpha p\bar{\theta} - k)}{\alpha p\bar{\theta} - k(1-\alpha)} & \alpha \ge \frac{k}{2}\left(1 + \sqrt{1 - \frac{4}{p\bar{\theta}}}\right), \end{cases} \tag{17}$$

where $\alpha_b \triangleq \frac{k}{2}(1+\sqrt{1-\frac{4}{p\bar{\theta}}}) < \alpha_a$ *and* $\beta^o(\alpha) \le \beta^*(\alpha)$. *The social optimal pair of asset composition and debt maturity structure* (α^o, β^o) *is:*

$$\alpha^o = \alpha^* = 1, \quad \beta^o = k\left(1 - \frac{k}{p\bar{\theta}}\right) < \beta^*. \tag{18}$$

Apparently, the manger's decision induces excessive roll-over risk and lowers the social welfare, which leaves room for welfare improvement by some regulation devices. In Fig. 2 we can find the gap between the optimal choices made by the manager (α^*, β^*) and the efficient outcome as represented by (α^o, β^o).

4. The impacts of NSFR

In this section, we introduce NSFR requirement and evaluate its impacts on manager's choices of asset composition and maturity structure, the bank's profit and social welfare. In our model, at date 0, the bank has 1 unit of asset and liability in total. Specifically, the bank has α share of long-term risky asset and $1 - \alpha$ share of cash on the asset side. Apparently, the required stable funding for cash is 0 and we denote the requirement stable funding factor for the risky asset as l. The bank also has β proportion of short-term debt and $1 - \beta$ proportion of long-term debt on the liability side. Denoting the available stable funding factor for short-term and long-term debt as $\tilde{h}$ and $\tilde{g}$ respectively, then $\tilde{g} > \tilde{h} \ge 0$ since long-term debt is a more stable funding source. By definition, we can write the NSFR requirement as: $\frac{\tilde{g}(1-\beta)+\tilde{h}\beta}{l\alpha} \ge 1$. Clearly, we can define $g = \tilde{g}/l$ and $h = \tilde{h}/l$, and the NSFR requirement becomes:

$$\frac{g(1-\beta) + h\beta}{\alpha} \ge 1 \tag{19}$$

The NSFR requirement is aimed to reduce the roll-over risk (liquidity risk) by restricting the joint effect of short-term debt and risky asset. As the proportion of short-term debt β increases, the maturity mismatch problem worsens and the manager has to increase cash holding $1 - \alpha$. On the other hand, as the proportion of risky asset α increases, the cash that can be used to cover the short-term debt repayment decreases and the manager has to use a higher proportion of stable (long-term) funding $1-\beta$. Obviously, if the bank uses only short-term debt to provide funds and invests in only risky asset, its roll-over risk gets maximized and thus the NSFR must be violated at $\alpha=\beta=1$, which means $h<1$; if the bank uses only long-term debt to provide funds, it faces no roll-over risk in our model and thus the NSFR must be satisfied at $\alpha=1$, $\beta=0$, which means $g\geq 1$. Under these two constraints, we can rewrite the NSFR requirement as:

$$\beta \leq \beta^r(\alpha) \triangleq 1-\frac{\alpha-h}{g-h}. \tag{20}$$

Actually, whether the regulation can achieve its aim and enhance social welfare is uncertain, and Example 1 shows this ambiguity.

Example 1. Assume that a bank has asset composition $\alpha=0.8$ and debt maturity structure $\beta=0.8$, and the relative weights for riskless and risky assets as required stable funding factors are respectively $g=1.2$ and $h=0.2$. Then according to expression (20), NSFR of this bank is:

$$\frac{1.2\times 0.2+0.2\times 0.8}{0.8}=50\%$$

Thus the bank violates the requirement $NSFR\geq 100\%$ due to too many short-term debts relative to its asset structure. If the manager wants to meet the regulation requirement, it can:

(1) reduce the proportion of short-term debt to 40%. This will make the bank exposed to less liquidity risk without hurting profitability since the asset composition maintains, and can also increase social welfare;
(2) reduce the investment in risky asset to 40%. This does NOT reduce bank's reliance on short-term debt, and further, it increases the bank's cash holding in a considerable proportion. Although a large amount of cash on the balance sheet can reduce liquidity risk, it is harmful to bank's profitability, and social welfare may be impaired.

Since there is incongruence of goals between the manager and bank owner, the manager may not always choose the optimal choices for the bank, and both cases may take place.

As King (2013) and Dietrich et al. (2014) documented, banks do not voluntarily choose the NSFR above the requirement, so we consider the case $\beta^*>\beta^r(\alpha^*)$, i.e., NSFR requirement is binding. Besides, a good regulation should not eliminate the social optimal outcome, and thus we allow $\beta^o\leq\beta^r(\alpha^o)$. Thus the NSFR requirement must satisfy the following conditions: (1) $g>1>h$; (2) $\beta^o(1)\leq\beta^r(1)<\beta^*(1)$. Two typical cases for regulation rules to affect bank manager's choices are shown in the following figures.

We can see that after regulation in enforce, (α^*, β^*) is excluded in Fig. 3 and the available set of pairs (α, β) the manager can choose is the region below the line $\beta=\beta^r(\alpha)$. Apparently this line intersects with line $\beta=1$ at $(h, 1)$ and with line $\alpha=1$ at $(1, \beta^r(1))$. Actually these two intersections also determine the regulation line itself. Thus if we denote $\beta^R\triangleq\beta^r(1)=\frac{g-1}{g-h}$, the pairs of (h, g) can be one-to-one mapped into pairs of (h, β^R). For convenience, from now on in this paper, we denote all regulation rules as (h, β^R) instead of (h, g), and now we can characterize the properties that regulation rules should have:

Regulation Requirement: (1) $h<1$; (2) $\beta^o(1)\leq\beta^R<\beta^*(1)$

In Fig. 3, we can see that the NSFR regulation line also intersects with line $\beta=\beta^*(\alpha)$ and the intersection must be unique since $\beta^*(\alpha)$ is concave in α. Denote this unique crossing point as (α_c, β_c), then $\alpha_c=h$ if $h<\alpha_a$ (Fig. 3a) and $\alpha_c>h$ if $h<\alpha_a$ (Fig. 3b). We know that the manager's payoff is maximized at $\beta^*(\alpha)$ for any given α and increases with α on line $\beta=\beta^*(\alpha)$, so the optimal pair of the manager under regulation must lie in the region below or on the line $\beta=\beta^*(\alpha)$ with $\beta\geq\beta_c$. We summarize state in the following lemma that the manager will choose the asset composition and debt maturity structure exactly on this regulation line.

Lemma 5. *Denote the asset composition and maturity structure chosen by the manager under NSFR requirement as (α^n, β^n), then it must satisfy $\beta^n=\beta^r(\alpha^n)$ and $\alpha^n\in[\alpha_c, 1]$.*

This lemma simplifies the manager's optimization problem under regulation of NSFR:.

$$\underset{\alpha\geq\alpha_c}{Max}\left\{M(\alpha,\beta^r(\alpha))=s\pi(\alpha,\beta^r(\alpha))-k\left(1+\int_{\tilde{\theta}(\alpha,\beta^r(\alpha))}^{\overline{\theta}}f(\theta)d\theta\right)\right\}. \tag{21}$$

To increase the social welfare, the most intuitive way is that the regulation makes the manager choose $(1, \beta^R)$. By regulation requirement, we know that $\beta^R\in[\beta^o, \beta^*)$, which means that regulation reduces the use of short-term debt with the asset composition unchanged. Thus if the manager chooses $(1, \beta^R)$) after regulation, social welfare is enhanced. The following proposition gives a simple way for this kind of regulation.

Proposition 3. *Under Assumptions A and B, if NSFR requirement (h, β^R) enforces $h=0$, then the manager will choose $(1, \beta^R)$ under the regulation.*

The regulation rule$h=0$ endows zero weight to short-term debt as available stable funding. Under this regulation, choosing short-term debt costs most for the manager to meet the requirement. The manager will reduce short-term debt as much as she can, i.e., $\beta=\beta^R$. And to restore her payoff, the manager will maximize the bank's investment in risky asset, i.e., $\alpha=1$. This result shows that a simple regulation rule $h=0$ can limit the manager's use of short-term debt, and reduce roll-over risk. It will hence increase the social welfare. With such a regulation rule, we can even achieve the social optimum, which is shown in Corollary 1 and Fig. 4.

Corollary 1. *If NSFR requirement $(h, \beta^R)=(0, \beta^o)$ is implemented, the manager will choose social optimal asset composition and maturity structure $(1, \beta^o)$.*

In reality, it may be difficult to implement the regulation rule of $h=0$. An important reason is that short-term debts may differ in maturities. Not like "cash" in our model, it is unrealistic to give all kinds of short-term debts a zero weight as available stable funding. Thus we have to find some more general regulation rules to enhance social efficiency. Still, we need to find conditions that the manager will choose $(1, \beta^R)$. To achieve this, we have to show that the payoff of manager at $(1, \beta^R)$ is no less than that at point (α_c, β_c). We have shown in the proof of Proposition 3 that $\lim_{\alpha\to 0}M(\alpha,1)<M(1,\beta^o(1))$, so $M(1,\beta^R)\in\left(\lim_{\alpha\to 0}M(\alpha,1),M(1,\beta^*(1))\right)$, and we know that the payoff of manager is increasing in α on the line $\beta=\beta^*(\alpha)$, which means that there exists unique $\alpha_d\in(0, 1)$, such that $M(1,\beta^R)=M(\alpha_d,\beta^*(\alpha_d))$. Apparently, $M(1, \beta^R)>M(\alpha_c, \beta_c)$ if and only if $\alpha_d>\alpha_c$. For any β^R, denote the h that just makes $\alpha_c(h,\beta^R)=\alpha^d(\beta^R)$ as $h_1(\beta^R)\in(0, 1)$, and we achieve the following proposition.

Proposition 4. *Under Assumptions A and B, for any $\beta^R\in[\beta^o, \beta^*)$, there exists unique $\tilde{h}(\beta^R)\in(0,1)$ satisfying $\tilde{h}(\beta^R)\leq h_1(\beta^R)$, and:*

(1) *if $h\in[0,\tilde{h}(\beta^R)]$, the asset composition and debt maturity structure combination chosen by the manager under NSFR requirement is $(1, \beta^R)$;*

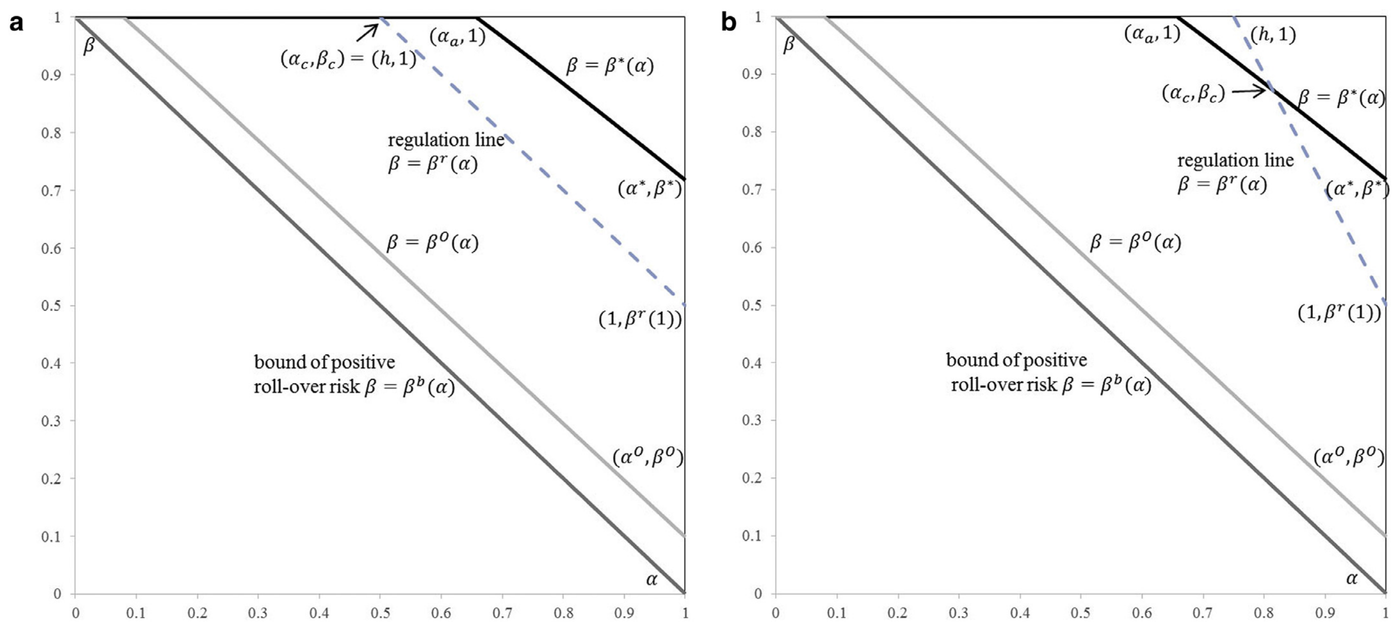

Fig. 3. (a) NSFR requirement ($g = 1.5,\ h = 0.5$), (b) NSFR requirement ($g = 1.25,\ h = 0.75$).

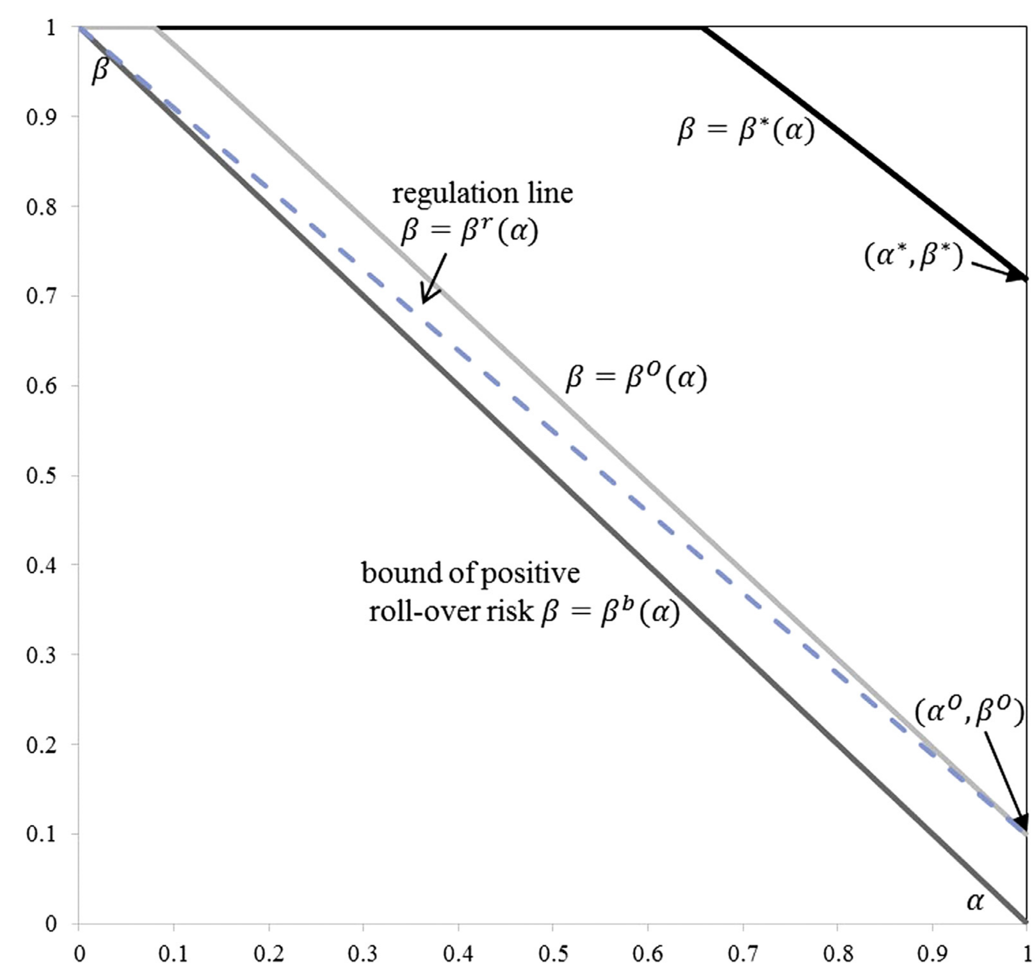

Fig. 4. NSFR requirement leading towards social optimum.

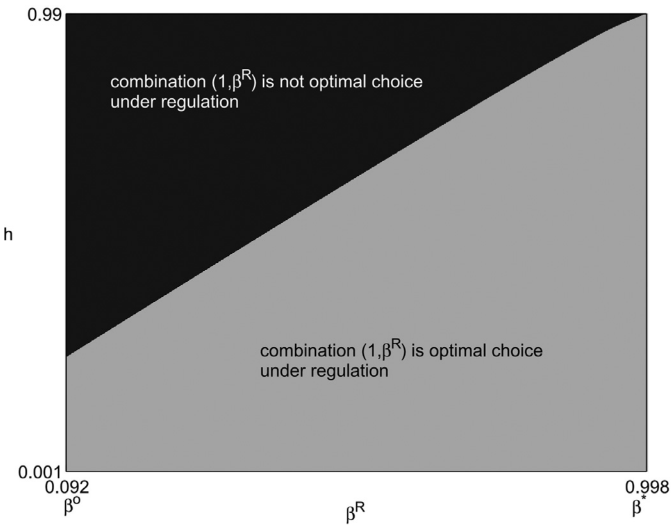

Fig. 5. The optimal choice for the manager under regulation.

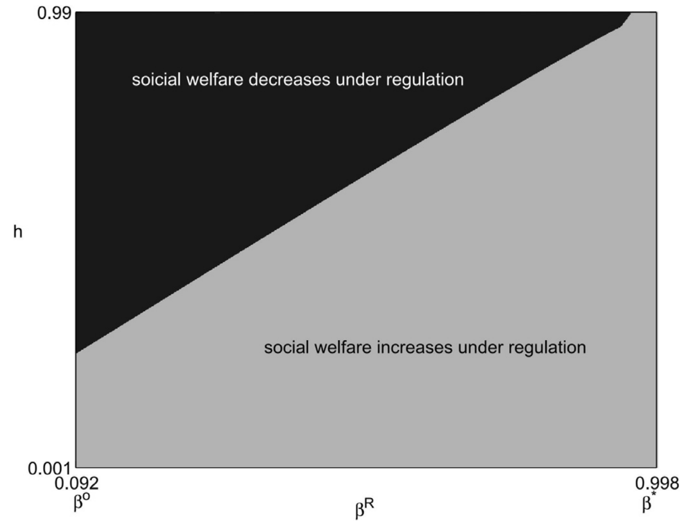

Fig. 6. The social welfare under regulation.

(2) *if* $h \in [\tilde{h}(\beta^R), 1)$, *the asset composition and debt maturity structure combination chosen by the manager under NSFR requirement lies on the line* $\beta = \beta^r(\alpha)$ *with* $\alpha \in [\alpha_c, 1)$.

This proposition shows that, if the social planner wants to enhance social welfare from the manager's un-regulated choices (1, β^*) to some (1, β^R) after regulation, there is an upper bound $\tilde{h}(\beta^R)$ of h for each β^R. This means that the regulation can achieve its goal if it is "strict" enough, i.e., it endows the short-term debt a low enough ASF factor h. The intuition of this proposition is similar with that of proposition 3. With this proposition, we know that if the regulation rule (h, β^o) is exerted, as long as $h \in [0, \tilde{h}(\beta^R)]$, the social optimum can be achieved. However, one the other hand, this proposition also implies that we should be cautious about the weights of available stable funding. If the weight of short-term debt is not strict enough $h > \tilde{h}(\beta^R)$, some extreme case can happen, i.e., (α_c, β_c) may be chosen by the manager, which reduces bank's profitability and is harmful to social welfare. The following proposition shows that NSFR regulation can enhance social welfare, conditional on it being sufficiently strict.

Proposition 5. *Under Assumptions A and B, for any regulation rule satisfying* $\beta^R \in [\beta^o, \beta^*)$ *and* $h \in [0, \tilde{h}(\beta^R)]$, *NSFR requirement will enhance social welfare. Particularly, the regulation rule* $\beta^R = \beta^o$ *and* $h \in [0, \tilde{h}(\beta^R)]$ *can achieve social optimal outcome.*

Figs. 5 and 6 simulate how the regulation affects the manager's choice and welfare with parameters $p = 0.5$, $\bar{\theta} = 8.01$, $k = 0.0945$, $s = 0.05$. The effects of all regulation rules $h \in [0, 1]$ (horizontal axis) and $\beta^R \in [\beta^o, \beta^*)$ (vertical axis) are shown. In Fig. 5, it is clear that there is an upper bound of h for each $\beta^R \in [\beta^o, \beta^*)$ such that the manager will choose asset composition and debt maturity structure (1, β^R) as optimal under regulation, as Proposition 4 presents. In Fig. 6, it is observed that in most cases (β^R is not close to β^*), if (1, β^R) is not chosen, then the real optimal choice for the manager under regulation will reduce social welfare in many cases. Further we can see that the upper bound of h is increasing in β^R. It means that the more social welfare that regulators want to improve, the more cautious they should be.

In this theoretical framework, we can also study the implications of NSFR requirement for bank's profit. Recall the profit function of the bank. Clearly this term is decreasing in β. The intuition is that: given the proportion of risky asset α, the bank's profit is directly determined only by one factor, the roll-over risk. High roll-over risk brings high liquidation cost and thus high debt face values, which reduces bank's expected profit. Thus the optimal maturity structure for the bank is $\beta = 0$. As the choice of manager's debt maturity structure decreases from β^* to β^R due to NSFR requirement, the *ex ante* profit of bank is raised. This means that NSFR regulation has a secondary effect on banks: NSFR regulation forces the manager to give up some private benefit and thus enhance the bank's profit as viewed from date 0.

Now we turn to the bank's *ex post* profit, which refers to the bank's expected profit given that they do not go bankruptcy: $\frac{\pi(\alpha,\beta)}{\int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} f(\theta)d\theta}$. This concept is important because it is consistent with what we observe in the sample. Data only records the performance of surviving banks, which means that the profits of banks in the sample measure the profits conditional on survival, or the *ex post* profit. Proposition 6 shows that the *ex post* profit will be reduced by NSFR requirement. The intuition is quite clear: as more short-term debt is used, roll-over risk is higher and the project's fundamental θ has to be higher to avoid early liquidation, which means that as long as the bank can roll over, it must have a higher project value and thus higher profit. The NSFR reduces the use of short-term debt from β^* to β^R. It also reduces the lower bound of fundamental $\tilde{\theta}(\alpha, \beta)$ and thus drives down the average profit of surviving banks.

Proposition 6. *Under Assumptions A and B, if NSFR requirement* (h, β^R) *satisfies* $h \in [0, \tilde{h}(\beta^R)]$ *and* $\beta^R \in [\beta^o, \beta^*)$, *then NSFR regulation will increase the bank's profit from the ex ante point of view, while decrease the bank's profit from the ex post point of view.*

This difference between *ex ante* and *ex post* profits is important for our understanding of the empirical results. We should be careful to interpret the relationship between NSFR requirement and banks' profits. In sum, the prediction of our model is that: NSFR reduces the probability of bank failures and also reduces the profits of survival banks.

5. Discussions

In this section, we relax some assumptions of the model. Our main results will not be affected by the relaxation of these assumptions.

5.1. The risky asset structure

In the model setup of this paper, we mentioned that θ realized at the interim state can be seen as a signal of the asset fundamental value: $\mathbb{E}(V \mid \theta) = p\theta$. However, as θ increases, the variance of the asset value perceived at date 1 ($Var(V \mid \theta) = p(1-p)\theta^2$) also increases. In this subsection, we will show that the effect of θ on asset's risk will not influence the results of our paper and θ can be an appropriate signal of the asset's fundamental value. We argue this from the following three aspects:

First, given any two realization values $\theta'' > \theta' \geq 0$, the conditional distribution $g(V|\theta')$ is first order stochastically dominated by $g(V|\theta'')$, which means for any investor with a non-decreasing function (including the risk neutral case for creditors as assumed in our paper), he/she will always have a higher expected utility conditional on θ'' than θ'. In this sense, θ can be considered as an appropriate signal of the asset's fundamental value.

Second, the asset's variance conditional on θ has no impact on the analysis of our paper. To see this, recall that the only role of θ as a useful piece of information at date 1 is to identify the roll-over condition for creditors, which is $\theta > \tilde{\theta} \triangleq \frac{\beta D_{0,1}-(1-\alpha)}{\alpha p}$, or $\alpha p\theta + (1-\alpha) > \beta D_{0,1}$. It means that if the expected value of the firm ($\alpha p\theta$ from the risky asset and $1-\alpha$ from the cash) can cover the face value of short-term debt $\beta D_{0,1}$, then roll-over can be achieved. Note that $p\theta$ is expected value of $\mathbb{E}(V \mid \theta)$ under a binomial distribution. Actually, for any distribution of $g(V|\theta)$ over $[0, +\infty)$, we prove in Appendix B in our paper that the roll-over condition is in the form of

$$\alpha\mathbb{E}(V \mid \theta) + (1-\alpha) > \beta D_{0,1} \tag{22}$$

So only $\mathbb{E}(V \mid \theta)$ matters for roll-over. As long as $\mathbb{E}(V \mid \theta)$ is increasing in θ, we can use θ as a sufficient statistics for the roll-over condition. Although $Var(V|\theta)$ also varies with θ, it has no effect on roll-over condition and thus the analysis of our model is not affected by the value of variance.

Third, if we specify an asset structure with its conditional variance independent of θ, all results of our paper can be maintained. A simple example is to assume the asset payoff V is θ with probability p and $\theta - \nu$ with probability $1-p$. Thus the variance of the asset payoff conditional on θ is $p(1-p)V^2$, independent of θ. With this setup, the roll-over condition is changed to $\theta > \tilde{\theta} \triangleq \frac{\beta D_{0,1}-(1-\alpha)}{\alpha p} + (1-p)\nu$ calculated from (22), and as long as $\alpha(1+(1-p)\nu) + \beta > 1$, there is positive roll-over risk for the firm. All break-even conditions and more importantly, the payoff function of the manager in (8) and social welfare in (9) are not affected. We analyze the optimal choice of the manager and welfare under regulation. The results show that the pattern of influences of regulation on manager's choice and welfare are not changed. The case with parameter combination $\nu = 0.1$, $p = 0.9$, $\bar{\theta} = 4$, $k = 0.05$, $s = 0.032$ is shown in Figs. 7 and 8, which is similar as those in Figs. 5 and 6[6].

Therefore, θ is an appropriate signal of the asset's value in the sense of being consistent with first order dominance. In addition, only expected value $\mathbb{E}(V \mid \theta)$ has a role in the analysis of our model, and the results are not changed if we specify an asset structure with a variance independent of θ. In the new version of our paper we explain clearly why it is reasonable for us to focus on the role of θ as an indicator of the asset's fundamental value.

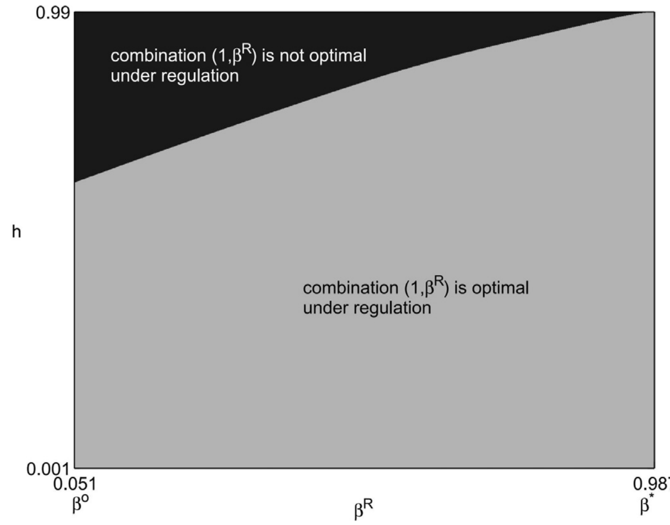

Fig. 7. The optimal choice for the manager under regulation ($\nu = 0.1$).

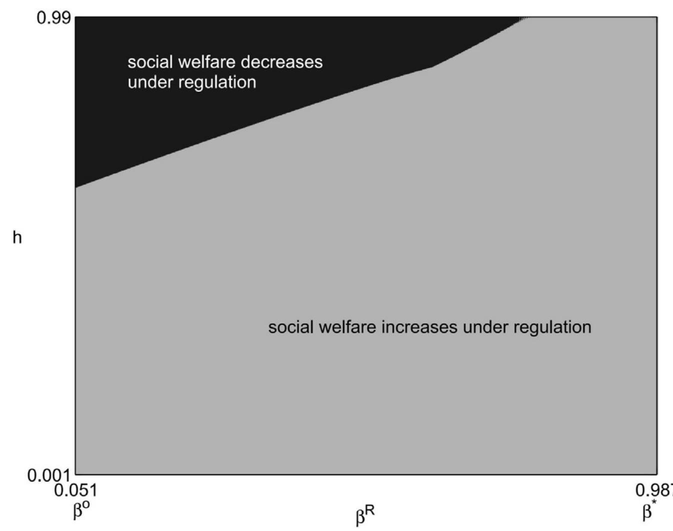

Fig. 8. The social welfare under regulation ($\nu = 0.1$).

5.2. Liquidation cost

In previous sections, we have assumed that liquidation value at date 1 for the asset is 0. This assumption is for tractability of our model. Alternatively, we can follow (Brunnermeier and Oehmke, 2013) and Huberman and Repullo (2013) to assume that the liquidation value is $\lambda \in [0, 1)$ fraction of asset's expected value at date 1, which is $\lambda p\theta$ in the framework of this paper. With this assumption, the break-even conditions for short-term and long-term creditors become

$$\int_0^{\tilde{\theta}(\alpha,\beta)} (\lambda\alpha p\theta + (1-\alpha)) f(\theta) d\theta + D_{0,1}\int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} f(\theta) d\theta = 1 \tag{23}$$

$$\begin{cases} \int_0^{\tilde{\theta}(\alpha,\beta)} (\lambda\alpha p\theta + (1-\alpha)) f(\theta) d\theta \\ + \int_{\tilde{\theta}(\alpha,\beta)}^{\hat{\theta}(\alpha,\beta)} \frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}} (\alpha p\theta + (1-\alpha)) f(\theta) d\theta \\ \int_{\hat{\theta}(\alpha,\beta)}^{\bar{\theta}} \left(p \cdot D_{0,2} + (1-p)\frac{D_{0,2}}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}}(1-\alpha)\right) f(\theta) d\theta \quad = 1 \end{cases} \tag{24}$$

[6] That the parameters $p, \bar{\theta}, k, s$ in Figs. 5 and 6 are all different from those of Figs. 7 and 8 is because the different values of ν lead to different parameter space which can satisfy the assumptions of our model. And in each case, we choose the best parameter combination to visualize our results.

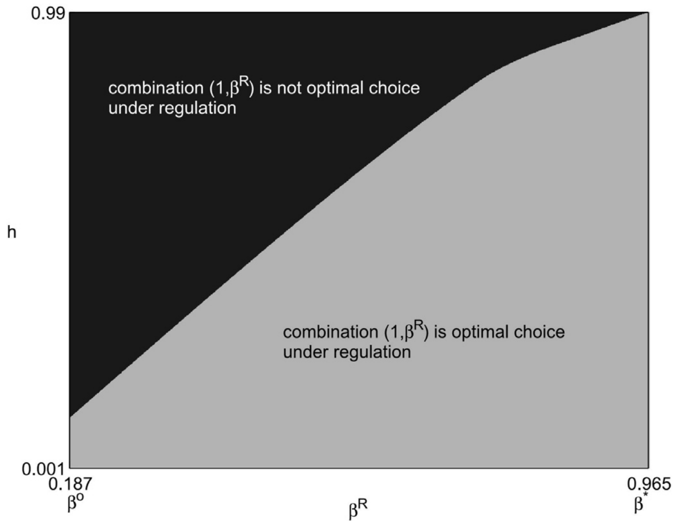

Fig. 9. The optimal choice for the manager under regulation ($\lambda = 0.5$).

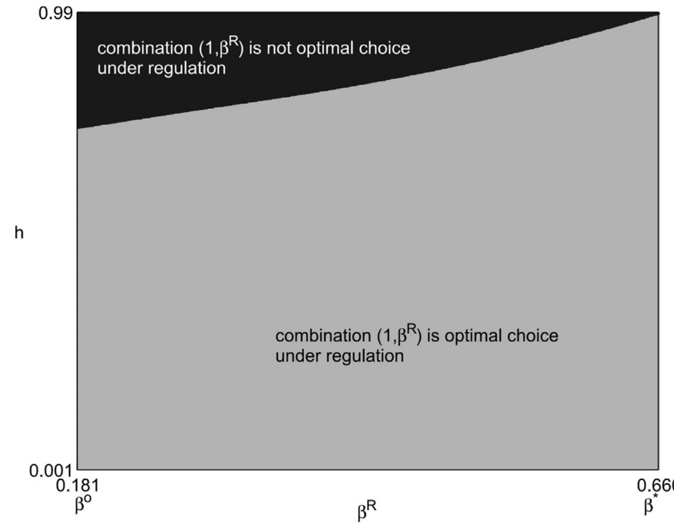

Fig. 11. The optimal choice for the manager under regulation ($\lambda = 0.99$).

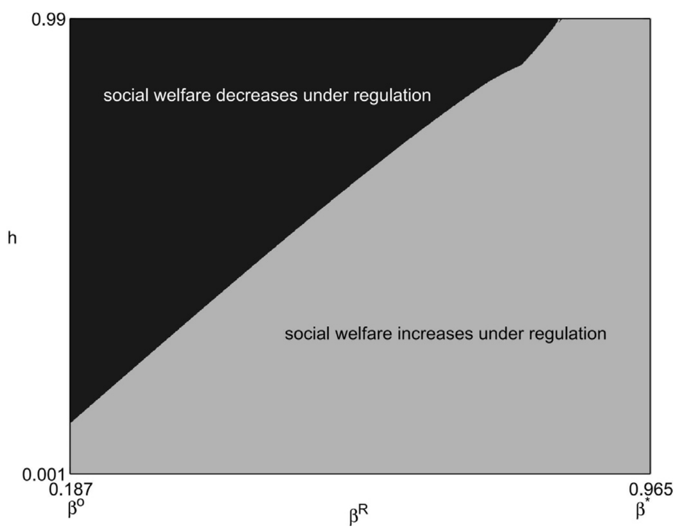

Fig. 10. The social welfare under regulation ($\lambda = 0.5$).

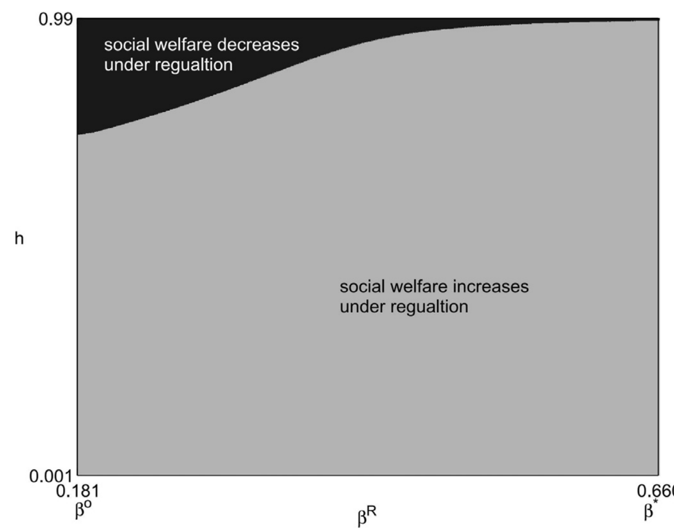

Fig. 12. The social welfare under regulation ($\lambda = 0.99$).

and the profit function changes to

$$\pi(\alpha,\beta) = \alpha\left(\lambda\int_0^{\tilde{\theta}(\alpha,\beta)} p\theta f(\theta)d\theta + \int_{\tilde{\theta}(\alpha,\beta)}^{\bar{\theta}} p\theta f(\theta)d\theta - 1\right) \tag{25}$$

while all other conditions and functions remain unchanged. More importantly, the roll-over condition is not related to the liquidation value, which implies that the main results of our paper are not affected by setting $\lambda > 0$. But with $\lambda > 0$, the roll-over cutoff value of θ at date 1 depend on λ:

$$\tilde{\theta}(\alpha,\beta) = \frac{\alpha p\bar{\theta} - (1-\alpha)(1-\beta) - \sqrt{\left((1+\beta-\lambda\beta)(1-\alpha)+\alpha p\bar{\theta}\right)^2 - 4\left(\beta - \frac{\lambda}{2}\beta^2\right)\left((1-\frac{\lambda}{2})(1-\alpha)^2 + \alpha p\bar{\theta}\right)}}{\alpha p(2-\lambda\beta)} \tag{26}$$

which is different from the case with $\lambda = 0$ as in Eq. (12). There is an extra term $2 - \lambda\beta$ in the denominator, which makes it rather difficult to obtain an explicit solution with clean properties of manager's payoff function.

However, we can provide numerical examples to show that our main results are not affected if we assume $\lambda > 0$. In Figs. 9 and 10 under parameter combination $\lambda = 0.5$, $p = 0.5$, $\bar{\theta} = 6.01$, $k = 0.01$, $s = 0.12$, we show that our results under this more general assumption exhibit similar patterns with Proposition 4 (Figs. 5 and 6). The pattern will not disappear even if we set $\lambda = 0.99$ ($p = 0.5$, $\bar{\theta} = 4.05$, $k = 0.00199$, $s = 0.2$) as in Figs. 11 and 12. So it is reasonable to believe that relaxing the assumption from $\lambda = 0$ to $\lambda \in [0, 1)$ will not change the results of our paper.

5.3. Leverage ratio

In this paper, there is an implicit assumption that there is no equity for the bank, i.e., the bank has an infinitely large leverage. Actually, relaxing this assumption will not change our main results. In this subsection, we introduce the case of finite leverage to discuss the effect of leverage ratio. To see this, assume that at date 0, the banker has $e \in (0, 1)$ unit of equity, then leverage is $L = 1/e \in (0, +\infty)$. Then the banker only has to fund $1 - e$ units of capital. This will change the break-even conditions of creditors, but in a simple way. Particularly, the roll-over condition becomes:

$$\theta > \tilde{\theta}(\alpha,\beta) = \frac{(1-\frac{1}{L})\beta D_{0,1}(\alpha,\beta) - (1-\alpha)}{p\alpha}. \quad (27)$$

Obviously, as leverage increases, roll-over risk for banker increases, and as $L \to +\infty$ we get back to the condition in (4). And since the asset value and management cost are both independent from the leverage ratio, we can conclude that the manager's problem is still given by (8) and first order condition for interior solution remains:

$$k = s\alpha p\tilde{\theta}(\alpha,\beta)$$

Apparently, β^* is decreasing in *L*. To understand this, we can observe from the above FOC. that in optimality, $\tilde{\theta}(\alpha,\beta)$ is constant given α. We know that both leverage and short-term debt increase roll-over risk. Thus faced with a high leverage, the manager has to choose a low level of short-term debt to avoid excessive roll-over risk. In this sense, leverage and short-term debt are substitutes. It is notable that although the proportion of short-term debt β^*decreases with *L*, the total use of short-term debt $(1-\frac{1}{L})\beta^*$ increases with *L*.

5.4. Seniority

Our model follows (Brunnermeier and Oehmke, 2013) by assuming that long-term creditors and short-term creditors have equal seniority. However, Diamond (1993) demonstrates that it is optimal for the firm to claim that short-term debt is senior to long-term debt, because it will increase the amount of debt that can be refinanced in the future. This implication was matched with data of leveraged buyouts. Eisenbach (2013) also assumes that short-term creditors get debt payment first. In fact, our result can easily be extended to the situation that short-term debt has seniority over long-term debt.

In our framework, if the short-term creditors have seniority, they will roll over only if the expected asset value can cover the cost of debt, which means $\beta D_{1,2}(\theta;\alpha,\beta) \le \alpha\theta + (1-\alpha)$. The break-even condition for roll-over creditors then becomes:

$$D_{0,1} = p\cdot D_{1,2}(\theta;\alpha,\beta) + \frac{(1-p)(1-\alpha)}{\beta} \quad (28)$$

The condition for existence of a solution to this equation is still: $\tilde{\theta}(\alpha,\beta) < \theta < \bar{\theta}$. Intuitively, if the roll-over creditors decide to lend money to the firm, they can set the face value of roll-over debt infinitely high to achieve the bank's whole asset value, as long as the long-term creditors do not have full seniority. This means the roll-over creditor has priority *de facto*, and thus roll-over condition will not be changed by creditors' seniority.

6. Conclusion

In this paper, we develop a theoretical framework in which the bank manager chooses asset composition and debt maturity structure. We model the incongruence of goals between the bank manager and the bank stakeholders by letting the bank manager to receive only a share of the bank's profit. We demonstrate that the bank manager's choices lead to socially inefficient outcomes, leaving room for welfare improvement in government regulation.

Within our theoretical framework, we discuss the impacts of the NSFR requirement on the bank manager's choices of asset composition and debt maturity structure, with consequences for the banks' profitability and on social welfare. We demonstrate that if short-term debt is given a sufficiently low weight in available stable funding, the NSFR can lower the use of short-term debt and thus reduce banks' exposure to roll-over risk. Social welfare can be enhanced under this set of conditions, but may be reduced when the short-term debt is given a sufficiently high weight. We also find that under the same set of conditions, the NSFR can increase the banks' probability of survival and unconditional expected profits (ex ante), because the constraint on the debt maturity structure alleviates the goal-incongruence problem between the bank owner and the manager. However, the NSFR will decrease the profitability of bank failures and the ex post profit of surviving banks.

The results of our paper are robust if we can assume an asset structure with the conditional variance on the interim information being a constant. Moreover, our theoretical framework can be shown to be useful in studying issues on the regulation of liquidity risks. It is possible to allow the liquidation value of the asset at date 1 to be related to the asset's fundamental value and continue to use our framework to analyze the impacts of bank regulation. It is possible to allow the bank to have equity on the liability side. It is also possible to allow the short-term debt to have seniority over the long-term debt. In the future, we hope to incorporate capital adequacy regulation and other regulation instruments into the theoretical framework to study the combined effect of various regulation schemes.

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Appendix A. Proofs of lemmas and propositions

Proof of Lemma 1. First we solve the break-even condition for short-term creditors. The explicit expression of condition (5) is as follows:

$$\beta D_{0,1}^2 - \left((1+\beta)(1-\alpha) + \alpha p\bar{\theta}\right)D_{0,1} + \alpha p\bar{\theta} + (1-\alpha)^2 = 0$$

Solving this equation, we have:

$$D_{0,1} = \frac{(1+\beta)(1-\alpha) + \alpha p\bar{\theta} \pm \sqrt{\left((1+\beta)(1-\alpha)+\alpha p\bar{\theta}\right)^2 - 4\beta\left(\alpha p\bar{\theta} + (1-\alpha)^2\right)}}{2\beta}$$

Since $\left((1+\beta)(1-\alpha)+\alpha p\bar{\theta}\right)^2 - 4\beta\left(\alpha p\bar{\theta}+(1-\alpha)^2\right) \ge (2(1-\alpha) + \alpha p\bar{\theta})^2 - 4\left(\alpha p\bar{\theta} + (1-\alpha)^2\right) = \alpha^2\left((p\bar{\theta})^2 - 4p\bar{\theta}\right)$, this term is non-negative and $D_{0,1}$ is a real number as long as Assumption A holds.

Then we solve the break-even condition for long-term creditors. Using Eqs. (2), (3), (5) to eliminate $D_{0,1}(\theta;\alpha,\beta)$ in Eq. (6) we get:

$$\left\{\begin{array}{l} \int_0^{\tilde{\theta}(\alpha,\beta)}(1-\alpha)f(\theta)d\theta + (\int_{\tilde{\theta}(\alpha,\beta)}^{\hat{\theta}(\alpha,\beta)}(\alpha p\theta + (1-\alpha))f(\theta))d\theta \\ \quad + \int_{\hat{\theta}(\alpha,\beta)}^{\bar{\theta}}(p\cdot(\beta D_{1,2}(\theta;\alpha,\beta) + (1-\beta)D_{0,2}) \\ \quad + (1-p)(1-\alpha))f(\theta)d\theta \end{array}\right. = 1$$

Thus solving Eq. (6) is equal to solving this equation. Define the left hand side of the equation as $L(D_{0,2}|D_{0,1},\beta)$, then

$$L(0|D_{0,1},\beta)=(\beta-1)D_{0,1}\int_{\tilde\theta(\alpha,\beta)}^{\bar\theta}pf(\theta)d\theta\leqslant 0$$

and

$$\frac{\partial L}{\partial D_{0,2}}=\int_{\hat\theta(\alpha,\beta)}^{\bar\theta}p\left((1-\beta)+\beta\frac{\partial D_{1,2}(\theta;\alpha,\beta)}{\partial D_{0,2}}\right)f(\theta)d\theta\geqslant 0$$

Note that $\hat\theta=\bar\theta$ for $D_{0,2}\to+\infty$, so we have

$$\begin{aligned}L(D_{0,2}=+\infty|D_{0,1})&=\alpha\left(\int_{\tilde\theta(\alpha,\beta)}^{\bar\theta}p\theta f(\theta)d\theta-1\right)\\&=\frac{\alpha}{2\bar\theta}\left[2\bar\theta\left(\frac{p\bar\theta}{2}-1\right)-\frac{1}{p}\left(\frac{\beta D_{0,1}-(1-\alpha)}{\alpha}\right)^2\right]\\&\geq\frac{\alpha}{2p\bar\theta}\left[2p\bar\theta\left(\frac{p\bar\theta}{2}-1\right)-\left(\frac{p\bar\theta-\sqrt{p\bar\theta(p\bar\theta-4)}}{2}\right)^2\right]\geq 0\end{aligned}$$

The last inequality holds because of Assumption A. In sum, we have $L(0|D_{0,1},\beta)\leq 0$, $L(D_{0,2}=+\infty|D_{0,1})\geq 0$ and $\frac{\partial L}{\partial D_{0,2}}\geqslant 0$. Thus there is positive solution to Eq. (6) for $\beta\in[0,1]$. □

Proof of Lemma 2. We need the condition that $\beta D_{0,1}\geq 1-\alpha$, or

$$\beta^2D_{0,1}^2-\left((1+\beta)(1-\alpha)+\alpha p\bar\theta\right)\beta D_{0,1}+\beta\left(\alpha p\bar\theta+(1-\alpha)^2\right)\geq 0$$

Rearranging this inequality, we get: $\beta\geq 1-\alpha$. □

Proof of Lemma 3. We can calculate that:

$$\frac{\partial\tilde\theta(\alpha,\beta)}{\partial\alpha}=\frac{(1-\beta)}{2p\alpha^2}+\frac{(1-\beta)^2(1-\alpha)+(1-\beta)\alpha p\bar\theta}{2p\alpha^2\sqrt{\left((1+\beta)(1-\alpha)+\alpha p\bar\theta\right)^2-4\beta\left(\alpha p\bar\theta+(1-\alpha)^2\right)}}\geq 0$$

$$\frac{\partial\tilde\theta(\alpha,\beta)}{\partial\beta}=\frac{1-\alpha}{2p\alpha}+\frac{(1-\beta)(1-\alpha)^2+\alpha(1+\alpha)p\bar\theta}{2p\alpha\sqrt{\left((1+\beta)(1-\alpha)+\alpha p\bar\theta\right)^2-4\beta\left(\alpha p\bar\theta+(1-\alpha)^2\right)}}>0.$$

□

Proof of Lemma 4. To solve the optimal maturity structure (given asset structure) explicitly, we only have to discuss the sign of the expression in the bracket in (13), since the terms outside the bracket is positive. Define

$$\Gamma(\alpha,\beta)\triangleq k-s\alpha p\tilde\theta(\alpha,\beta)$$

Apparently $\Gamma(\alpha,\beta^b(\alpha))>0$ and $\Gamma_\beta(\alpha,\beta)<0$, so we only need to discuss the property of $\Gamma(\alpha,1)$, where:

$$\Gamma(\alpha,1)=k-\frac{2s\alpha\bar\theta}{\bar\theta+\sqrt{\bar\theta^2-\frac{4\bar\theta}{p}}}$$

To make interior solution possible, we need $\Gamma(1,1)<1$, or $k(1+\sqrt{1-\frac{4}{p\bar\theta}})<2s$. Thus for $\alpha\leq\frac{k}{2s}(1+\sqrt{1-\frac{4}{p\bar\theta}})$, $\beta=1$ is optimal and for $\alpha\geq\frac{k}{2s}(1+\sqrt{1-\frac{4}{p\bar\theta}})$, the optimal β is the solution of $\Gamma(\alpha,\beta)=0$, which is:

$$\beta=\frac{\left(\frac{k}{s}+(1-\alpha)\right)\left(\alpha p\bar\theta-\frac{k}{s}\right)}{\alpha p\bar\theta-\frac{k}{s}(1-\alpha)}=1-\frac{1}{p\bar\theta}\frac{\alpha^2-\frac{k}{s}\alpha+\frac{k^2}{s^2}\frac{1}{p\bar\theta}}{(p\bar\theta+\frac{k}{s})\alpha-\frac{k}{s}}$$

Clearly, for $\alpha\geq\frac{k}{2s}(1+\sqrt{1-\frac{4}{p\bar\theta}})$, we have

$$\begin{aligned}\frac{d\beta}{d\alpha}&=-\frac{1}{p\bar\theta}\frac{\alpha^2(p\bar\theta+\frac{k}{s})-2\alpha\frac{k}{s}+\frac{k^2}{s^2}-(p\bar\theta+\frac{k}{s})\frac{k^2}{s^2}\frac{1}{p\bar\theta}}{\left((p\bar\theta+\frac{k}{s})\alpha-\frac{k}{s}\right)^2}\\&\geq-\frac{1}{p\bar\theta}\frac{k^4}{s^4}\sqrt{1-\frac{4}{p\bar\theta}}\frac{(p\bar\theta+\frac{k}{s})\frac{1}{2}(1+\sqrt{1-\frac{4}{p\bar\theta}})-1}{\left((p\bar\theta+\frac{k}{s})\alpha-\frac{k}{s}\right)^2}\geq 0\end{aligned}$$

$$\frac{d^2\beta}{d\alpha^2}=-\frac{1}{p\bar\theta}\frac{k^2}{s^2}\frac{1+\frac{k}{p\bar\theta}(p\bar\theta+k)}{\left((p\bar\theta+\frac{k}{s})\alpha-\frac{k}{s}\right)^3}<0.$$

□

Proof of Lemma 4. From the proof of Lemma 4, we know that given any α, $\frac{\partial M(\alpha,\beta)}{\partial\beta}>0$ for $\beta<\beta^*(\alpha)$, which means manager's payoff reaches maximum at $\beta^r(\alpha)$ for any given α. □

Proof of Proposition 3. First derivative gives:

$$\frac{dM(\alpha,\beta^r(\alpha))}{d\alpha}=s\left(\int_{\tilde\theta(\alpha,\beta^r(\alpha))}^{\bar\theta}p\theta f(\theta)d\theta-1\right)+\frac{1}{\bar\theta}\left[k-s\alpha p\tilde\theta(\alpha,\beta^r(\alpha))\right]\frac{d\tilde\theta(\alpha,\beta^r(\alpha))}{d\alpha}$$

$$\begin{aligned}\frac{d^2M(\alpha,\beta^r(\alpha))}{d\alpha^2}&=-\frac{sp}{\bar\theta}\frac{d\tilde\theta(\alpha,\beta^r(\alpha))}{d\alpha}\left(2\tilde\theta(\alpha,\beta^r(\alpha))+\alpha\frac{d\tilde\theta(\alpha,\beta^r(\alpha))}{d\alpha}\right)\\&\quad+\frac{1}{\bar\theta}\left[k-s\alpha p\tilde\theta(\alpha,\beta^r(\alpha))\right]\frac{d^2\tilde\theta(\alpha,\beta^r(\alpha))}{d\alpha^2}\end{aligned}$$

where

$$\begin{aligned}\frac{d\tilde\theta(\alpha,\beta^r(\alpha))}{d\alpha}&=-\frac{1}{g}\frac{\partial\tilde\theta(\alpha,\beta^r(\alpha))}{\partial\beta}+\frac{\partial\tilde\theta(\alpha,\beta^r(\alpha))}{\partial\alpha}\\&=-\frac{1}{2pg}\left(\frac{1}{\sqrt{1-\frac{4p\bar\theta(1-\frac{1}{g})}{(p\bar\theta-\frac{1}{g}(1-\alpha))^2}}}-1\right)<0\end{aligned}$$

and obviously $\frac{d\tilde{\theta}^2(\alpha,\beta^r(\alpha))}{d\alpha^2} > 0$. With

$$2\tilde{\theta}(\alpha, \beta^r(\alpha)) + \alpha\frac{d\tilde{\theta}(\alpha, \beta^r(\alpha))}{d\alpha}$$
$$\geq \tilde{\theta}(\alpha, \beta^r(\alpha))\left(2 - \frac{1}{\sqrt{4gp\bar{\theta} + g^2 p\bar{\theta}\left(p\bar{\theta} - 4\right)}}\right) \geq 0$$

we have $\frac{d^2 M(\alpha,\beta^r(\alpha))}{d\alpha^2} > 0$. Thus $M(\alpha, \beta^r(\alpha))$ is convex in α and we only need to compare $M(1, \beta^R) \geq M(1, \beta^o(1)) = s\left(\frac{p\bar{\theta}}{2} - 1\right) + \frac{(2-s)k}{4}\left(1 - \sqrt{1 - 4k\frac{p\bar{\theta} - k}{p^2\bar{\theta}^2}}\right) - 2k$ and $\lim_{\alpha\to 0} M(\alpha, \beta^r(\alpha)) = \frac{k}{2}\left(1 - \sqrt{1 - \frac{4}{p\bar{\theta}}}\right) - 2k$. From Assumption B that $\lim_{\alpha\to 0} M(\alpha, \beta^r(\alpha)) < s\frac{1 - \sqrt{1 - \frac{4}{p\bar{\theta}}}}{1 + \sqrt{1 - \frac{4}{p\bar{\theta}}}} < s\left(\frac{p\bar{\theta}}{2} - 1\right) < M(1, \beta^R)$, so the optimal choice of α under regulation is $\alpha = 1$. □

Proof of Proposition 4. For any β^R, we can find all pairs of (α, β) below the line $\beta = \beta^*(\alpha)$ and satisfying $M(\alpha, \beta) = M(1, \beta^R)$. Actually, it's a curve starting from $(\alpha_d, \beta^*(\alpha_d))$ to $(1, \beta^R)$. Denote this curve as $\beta = \beta^e(\alpha; \beta^R)$ ($\alpha \in [\alpha_d, 1]$), and it is decreasing in α. To see this, from $M(\alpha, \beta^e(\alpha; \beta^R)) = M(1, \beta^R)$, we get

$$\frac{dM(\alpha, \beta^e(\alpha; \beta^R))}{d\alpha} = \begin{cases} s\left(\int_{\tilde{\theta}(\alpha,\beta^e(\alpha;\beta^R))}^{\bar{\theta}} p\theta f(\theta)d\theta - 1\right) \\ +\frac{1}{\bar{\theta}}\left[k - s\alpha p\tilde{\theta}(\alpha, \beta^e(\alpha; \beta^R))\right]\left[\frac{\partial\tilde{\theta}(\alpha,\beta^e(\alpha;\beta^R))}{\partial\alpha}\right. \\ \left. + \frac{\partial\tilde{\theta}(\alpha,\beta^e(\alpha;\beta^R))}{\partial\beta}\frac{d\beta^e(\alpha;\beta^R)}{d\alpha}\right] \end{cases} = 0$$

With all other terms positive, we can get $\frac{d\beta^e(\alpha;\beta^R)}{d\alpha} < 0$ and further $\frac{d\beta^e(\alpha;\beta^R)}{d\alpha}\Big|_{\alpha=1,\beta=\beta^R} > 1 - \beta^R$. From Proposition 3, we can see that for any β^R, the regulation line $(0, \beta^R)$ is below the curve $\beta = \beta^e(\alpha; \beta^R)$ except the point $(1, \beta^R)$, so there must exist $\tilde{h}(\beta^R)$ such that $\beta = \beta^e(\alpha; \beta^R)$ has no intersection with regulation line (h, β^R) if $h < \tilde{h}(\beta^R)$ and intersects with regulation line (h, β^R) if $h > \tilde{h}(\beta^R)$. In the former case, the regulation line is below $\beta = \beta^e(\alpha; \beta^R)$ except the point $(1, \beta^R)$, so $(1, \beta^R)$ is optimal; in the latter case, some points on the regulation line is above $\beta = \beta^e(\alpha; \beta^R)$, and those points are strictly better than the point $(1, \beta^R)$, which means the point $(1, \beta^R)$ cannot be optimal. Apparently, if $h > h_1(\beta^R)$, at least point (α_c, β_c) is strictly better than the point $(1, \beta^R)$, so $\tilde{h}(\beta^R) \leq h_1(\beta^R)$. □

Proof of Proposition 6.

$$\frac{d\left(\frac{\pi(\alpha,\beta)}{\int_{\tilde{\theta}(1,\beta)}^{\bar{\theta}} f(\theta)d\theta}\right)}{d\beta} = \frac{-p\tilde{\theta}(1, \beta) + \frac{p}{2\bar{\theta}}\tilde{\theta}^2(1, \beta) + \frac{p}{2}\bar{\theta} - 1}{\left(\int_{\tilde{\theta}(1,\beta)}^{\bar{\theta}} f(\theta)d\theta\right)^2}\frac{1}{\bar{\theta}}\frac{d\tilde{\theta}(1, \beta)}{d\beta}$$
$$> 0. \qquad \square$$

Appendix B. Derivation of roll-over condition

At date 1, the break-even condition for the roll-over creditors is:

$$\begin{cases} \int_{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}}^{+\infty} D_{1,2}(\theta; \alpha, \beta)g(V \mid \theta)dV \\ + \int_0^{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}} \frac{D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}} \\ \times(\alpha pV + (1-\alpha))g(V \mid \theta)dV \end{cases} = D_{0,1}$$

where $D_{1,2}(\theta; \alpha, \beta)$ is the face value of roll-over debt which needs to be solved. Define the left hand side of the equation as:

$$N(D_{1,2}(\theta; \alpha, \beta)) = \begin{cases} \int_{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}}^{+\infty} D_{1,2}(\theta; \alpha, \beta)g(V \mid \theta)dV \\ + \int_0^{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}} \frac{D_{1,2}(\theta;\alpha,\beta)}{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}} \\ \times(\alpha pV + (1-\alpha))g(V \mid \theta)dV \end{cases}$$

Then $N(0) = 0 < D_{0,1}$ and

$$N'(D_{1,2}(\theta; \alpha, \beta)) = \begin{cases} \int_{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}}^{+\infty} g(V \mid \theta)dV \\ + \int_0^{\frac{\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2}-(1-\alpha)}{\alpha}} \frac{(1-\beta)D_{0,2}}{(\beta D_{1,2}(\theta;\alpha,\beta)+(1-\beta)D_{0,2})^2} \\ \times(\alpha pV + (1-\alpha))g(V \mid \theta)dV \end{cases} > 0$$

So for there existing a solution to $N(D_{1,2}(\theta; \alpha, \beta)) = 0$, we need $N(+\infty) > D_{0,1}$, which means

$$N(+\infty) = \int_0^{+\infty} \frac{\alpha pV + (1-\alpha)}{\beta} g(V \mid \theta)dV > D_{0,1}$$

or $\alpha\mathbb{E}(V \mid \theta) + (1-\alpha) > \beta D_{0,1}$. □

References

Basel Committee on Banking Supervision, 2011. Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems. Bank of International Settlement.

Basel Committee on Banking Supervision, 2014. Basel III: The Net Stable Funding Ratio. Bank of International Settlement.

Brunnermeier, M., 2009. Deciphering the liquidity and credit crunch 2007–2008. J. Econ. Perspect. 23 (1), 77–100.

Brunnermeier, M., Oehmke, M., 2013. The maturity rat race. J. Finance 68 (2), 483–521.

Diamond, D., 1991. Debt maturity structure and liquidity risk. Q. J. Econ. 106 (3), 709–737.

Diamond, D., 1993. Seniority and maturity of debt contracts. J. financ. Econ. 33 (3), 341–368.

Diamond, D., He, Z., 2014. A theory of debt maturity: the long and short of debt overhang. J. Finance 69 (2), 719–762.

Diamond, D., Rajan, R., 2009. The credit crisis: conjectures about causes and remedies. Am.Econ. Rev. Pap. Proc. 99 (2), 606–610.

Dietrich, A., Hess, K., Wanzenried, G., 2014. The good and bad news about the new liquidity rules of basel III in western european countries. J. Bank. Financ. 44 (1), 13–25.

Eisenbach, T., 2013. Rollover Risk as Market Discipline: A Two-Sided Inefficiency. Federal Reserve Bank of New York. Staff Reports 597.

Hellwig, M., 2009. Systemic risk in the financial sector: an analysis of the subprime–mortgage financial crisis. De Economist 157 (2), 129–207.

Huberman, G., Repullo, R., 2013. Moral hazard and debt maturity. Working Paper.

King, M., 2013. The basel III net stable funding ratio and bank net interest margins. J. Bank. Financ. 37 (11), 4144–4156.